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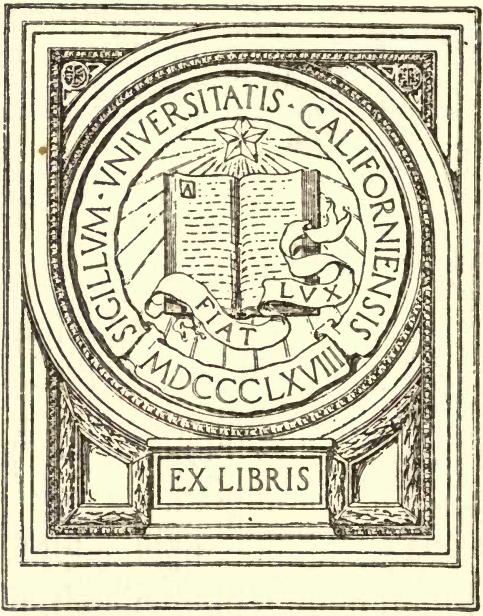
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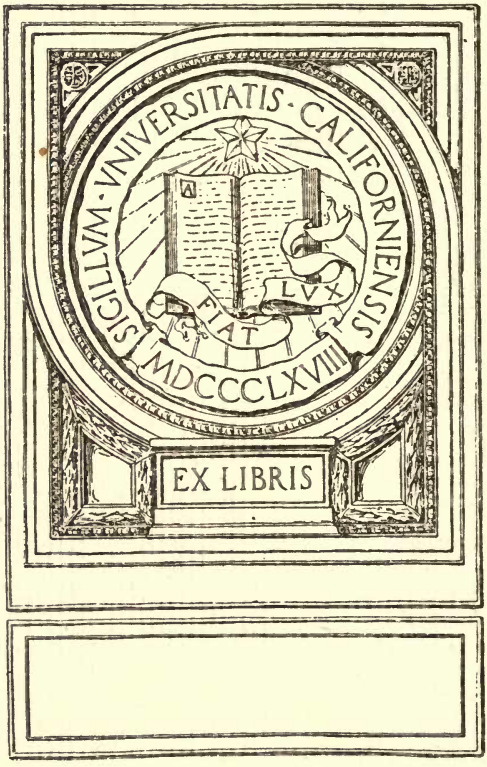
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A TEXT-BOOK

OF

CHEMICAL ARITHMETIC

BY

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PREFACE.

THIS book is designed especially for the use of students of quantitative analysis, many of whom, even after having taken extensive courses in higher mathematics, show little ability to solve simple chemical problems. Certain portions of the work are suitable also for the use of those who are studying elementary chemistry.

The subject is presented, as far as possible, in the simplest arithmetical manner, use being made of reasoning rather than rules and mathematical formulas. It must be admitted that arithmetic is very difficult for those who depend chiefly upon memory, but it has the advantage of being a means of developing the reasoning-power, which is undoubtedly a higher mental faculty than memory, and is also a valuable attainment as a preparation for a career in chemistry.

The first part of the book, dealing with approximate numbers, has been introduced on account of the poor judgment displayed by many students in carrying

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out decimals in the results of their work. The very useful methods of abbreviated multiplication and division are introduced here for the benefit of those who may not be familiar with them.

The subject matter has been carefully classified for the sake of convenience for reference· but it is to be strongly recommended that the problems given in the class-room be sufficiently varied in their forms and in the principles involved in them, so that the students may be forced to abandon the use of pure memory for their solution. It may be suggested that problems be solved in the class-room with the aid of the author's "Tables for Chemical Calculations," or other logarithmic tables.

The author is indebted to Professor Percy F. Smith, whom he has consulted in regard to some points of mathematics; also to Mr. Carl O. Johns, who has rendered valuable assistance, particularly in the verification of the answers to problems; and he is particularly grateful to Professor Harry W. Foote, who has made many valuable suggestions which have led to important modifications in the book.

H. L. W.

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CHEMICAL ARITHMETIC.

PART I.

APPROXIMATE NUMBERS.

1. Introduction. The numbers dealt with in chemical calculations, as far as they depend upon observations, such as the results of weighing and measuring, as well as atomic weights and other experimental constants, are always approximate ones. No matter how accurately an observation may be made, there must be a figure in the recorded result, expressed in the decimal system, beyond which nothing is known.

For instance, if the weight 1.2345 g is taken on a balance which is sensitive only to the nearest tenth of a milligram, the value of the true figure following the 5 is unknown, and it is uncertain whether it is positive or negative; but if the last figure observed is the nearest one to the truth, it is evident that the unknown figure following it must be less than ± 5.

If the weight under consideration had been taken with a less sensitive balance, or with less accurate weights, the last figure might be uncertain; but if its uncertainty was slight, for instance, if the figure did not probably vary more than one or two units from the truth, it is obvious that it should be retained, for the observed weight would be less accurate if this figure were omitted. On the other hand, if the weight had been taken in such a way that the figure before the last was uncertain, it would be reasonable to omit the last figure, with the customary addition of a unit to the preceding one if the omitted figure was as large as 5.

In order to distinguish an uncertain last figure from one that is presumably exact, the uncertain figure may be underlined; thus, 1.234$\underline{5}$ g indicates that the $\underline{5}$ is an approximation.

2. Apparent Errors. It is evident that an error must be involved in any approximate number. If nothing is known concerning the way in which the number has been obtained, it may be assumed, for the purposes of calculation, to be correct as far as the figures are given, and hence the last figure may be supposed to be the nearest one to the truth. If this is assumed to be the case, the error in the number cannot be greater than five units in the place beyond the last figure.

These five units in the place beyond the last figure of an approximate number indicate the *maximum*

apparent error of the number. Some examples are as follows:

Number.....	23.612	152.3	.8721	.00134
Max. app. error..	±.0005	±.05	±.00005	±.000005

An error which is actually a maximum one would be expected to occur but rarely; but, since the assumption that the last figure of an approximate number is the nearest one to the truth is unwarranted in many cases, this maximum apparent error may be used in the consideration of the very uncertain values of the errors in such numbers, and it will be the one to be understood when apparent errors are referred to in the discussions that follow.

3. Accidental Errors. Errors depending upon mistakes in observations, imperfect processes, etc., occurring in chemical data, may be much more serious than the apparent errors of the numbers. These accidental errors are not usually evident in the data presented for problems, but they are so important in practice that they should receive some consideration here.

Large accidental errors may usually be detected by repeating an observation or determination. Hence it is customary for chemists to make two or more analyses of a substance in most cases, and the average of satisfactorily agreeing determinations is taken as the most probable result. However, the agreement of two or more results obtained under the same conditions by the same method does not prove the absence of errors inherent

in the method, which are likely to be nearly constant and to be much more serious than any errors due to weighing or measuring. On this account, the calculation of the mathematical "probable error," which is based upon the variations of a series of independent observations, and which gives no indication of constant errors, is of little use in connection with chemical work, and it will not be discussed here.

4. Absolute and Proportional Errors. When the absolute value of an error is given, such as .001 g. .05 ccm, etc., the magnitude of the number which it affects is not considered; but, for the purposes of calculation, it is of the utmost importance to know the relations of errors to the numbers to which they belong. The proportional error is expressed in terms of the number affected by it. For instance, the weight .0113 g, with an error of .00005 g, has an error of five parts in 1130, or one part in 226; while the same absolute error in the weight 1.1300 g is one part in 22 600. It is useless, however, to express such uncertain values with the precision that has just been used, hence the proportional errors just mentioned would be given ordinarily as one part in about 200 and 20 000.

When the proportional error of a number is given, the absolute error may be calculated very simply. For example, if the number 15.35 has an error of one part in 2000, it is only necessary to divide the number by 2000 to find that the absolute error is about .008.

5. The Addition and Subtraction of Approximate Numbers. With numbers having errors of the same order, these processes require no discussion here, except the statement that the usual apparent errors may generally be assumed for the results of the operations, because the errors of any two of the numbers may have the same or opposite algebraic signs with equal probability, and, therefore, are just as likely to increase as to diminish one another.

When numbers having errors of different orders are added or subtracted, however, the results frequently contain figures that may be discarded as entirely without significance. For example, let us add the following numbers which are assumed to have apparent errors only:

$$\begin{array}{r} 112.4 \\ 70.90 \\ 36.032 \\ \hline 219.3\underline{32} \end{array}$$

The underlined figure is uncertain, because the true value of 112.4 is unknown beyond the first decimal place, and hence this figure in the sum has a maximum error of five units derived from the apparent error of the number just mentioned. It is not best to omit this uncertain figure, at least for the purpose of further, careful calculation, but the last figure may be discarded and the sum may be expressed as $219.3\underline{3}$.

6. The Multiplication and Division of Approximate Numbers. When several approximate numbers are used for multiplication and division, each error in the numbers used affects the result in the same proportion as it affected the original number. For example, if the number 200 with an error of 2, or one part in 100, is multiplied or divided by another number, say by 10, both the number, 200, and its error, 2, will be multiplied or divided by 10; hence the result of multiplication will be 2000 with an error of 20, and that of division will be 20 with an error of .2, so that in each case the error of the result is one part in 100, like the proportional error of the original number. Again, if the number 990 is divided by 10, by $10+1$, and by $10-1$, in order to show the effect of an error of one part in 10 in the divisor, the quotients are 99, 90, and 110, and it is found that 90 is smaller than 99 by one-tenth of its value, while 110 is larger than 99 in the same proportion, so that it is evident that a variation in the divisor produces the same proportional variation in the quotient, but in an opposite direction.

The largest proportional error among several that may affect the result of a multiplication or division, or of both, of several members, is the only one that usually need be considered. For when the others are decidedly smaller they may be disregarded in connection with these inexact values. When there are two, practically equal, larger errors, they are just as liable to have opposite algebraic signs, and thus coun-

teract each other, as to have the same sign, and thus double one another, so that the effect of one of them is the average result to be expected. When there are more than two larger errors of nearly equal value, these errors and their results may be regarded as acting in pairs, thus showing that their effect is essentially like that of the preceding case. This last case, however, is not a common one in chemical calculations.

7. The Significant Figures in the Results of Calculations. Beyond the figures that are presumably certain in the results of calculations, there is generally one more or less uncertain figure which is of some importance, because the truth will usually be more nearly approached by retaining than by omitting it. The apparently certain figures, together with the uncertain one, comprise the significant figures of a result.

It is obvious that all the significant figures of a result should be preserved, for, otherwise, an error would be introduced from the calculation itself, but it is manifestly absurd to retain figures that have no significance.

The last significant figure of a result, that is, the uncertain one, may be found from the greatest proportional error, either apparent or assumed, in the numbers from which the result is obtained, by the application of the principles set forth in the preceding section.

For example, if the number .0101 has been used in obtaining the result .00124685, and if the maximum

apparent error of .0101 is assumed, it is to be seen, since the result has a derived error of one part in about 200, that the figure 6 may have an error of about six units, and is consequently the last significant figure, so that the result should be expressed as .001247. Another way of finding the derived error is to notice that 124 is about one-fourth greater than 101, and hence it must have an absolute error about one-fourth larger than the latter, or about .6, if the apparent error .5 is assumed for 101. In comparing numbers in this way their decimal points are left out of consideration.

As another example, if the number .3261 has the largest proportional error among the numbers used to produce the result 65.8742, it is obvious that the fifth figure in the result will have about twice the uncertainty of the unknown, fifth figure of .3261; hence, if the maximum apparent error is assumed for the latter, an error of ten units in the fifth figure, or of one unit in the fourth figure, is the derived error of the result, which should be expressed as 65.87.

As a final example, let us consider the result, .07151. . . , of the operation indicated by $\frac{.0123 \times 2.567}{1.207 \times .3660}$, assuming maximum apparent errors to be present. The number .0123 is the one having the largest proportional error; hence, since 715 is nearly six times 123, there will be a derived maximum error of about three units in the 5, so that the result should be expressed as .0715. If an error of two units in the last figure of .0123 had

been assumed, the result would be .07$\underline{2}$ with an error of a little over one unit in the uncertain figure.

It should be noticed that the last significant figure —the uncertain one—may be often omitted in practical calculations, particularly when the final result is reached and the uncertainty in the figure is a large one. It is very evident that a figure with an error approaching ten units is far less worthy of preservation than one with an uncertainty not much greater than one unit. Decision in regard to this matter must be left to the judgment of the operator, and this should depend upon the object of the calculation, and upon the probable accuracy of the data employed as compared with their apparent errors. The main point to be insisted upon is the preservation of all the figures that are presumably reliable, and the omission of figures having no significance.

In the more practical parts of this book that follow this somewhat theoretical one, the course just outlined will be followed in giving answers to examples, and no attempt will be made to underline in all cases last figures that may be somewhat uncertain, although this will be done in some instances.

PROBLEMS.

1. What are the maximum apparent errors of 164.23 and .0127, expressed in absolute terms?

2. What are the maximum apparent errors of 164.23 and .0127, expressed proportionally?

3. If the absolute error in 6.142 is .005, what is the proportional error?

4. If the proportional error in 6.142 g is one part in 5000, what is the absolute error?

5. Add the numbers 63.6, 32.06, 64.0000, and 90.080, assuming maximum apparent errors.

6. Assuming maximum apparent errors, what is the result of 14.06×2.1380?

7. Assuming that the last figure of each number has an error of two units, what is the result of $14.0\underline{6} \div 2.729\underline{8}$?

8. Assuming maximum apparent errors, what is the result of $.4866 \div 19.0$?

9. Assuming maximum apparent errors, what is the result of $\frac{.27298 \times .50}{1.0185}$?

10. Assuming the underlined figures to be uncertain to the extent of two units, what is the result of $.48\underline{7} \div .51\underline{1}$?

11. Assuming that the number .167$\underline{8}$ has an uncertainty of nine units in the last figure, what would be the significant figures in a result, 42.7834, etc., derived from it by multiplication?

8. Relations between Errors in Weighing and the Accuracy of Analytical Results. If an error is made in weighing a substance to be analyzed, it will affect any result obtained in the analysis in the some proportion in which it affected the orignal substance. For example, suppose a substance to be weighed with

an error of one part in 1000. Anything determined in it will then be one part in 1000 too large or too small on account of this error. The absolute errors thus caused, however, will vary with the amounts determined; thus, one one-thousandth part of 10% is .01%, while of 2.5% it is .0025%. Errors in weighing the original substance, therefore, have a small absolute effect when the amount of substance determined is small.

Since percentages depend upon the weight of the substance analyzed, this weight determines the absolute errors caused by inaccuracies in weighing the substances determined. For example, 1.0000 g of substance is used for analysis, and .3000 g, .1000 g, and .0200 g of its constituents are weighed with an error in each case of .0002 g. This error amounts to .02% of the original substance, so that this is the absolute error in each case.

When a substance to be determined is not weighed as such, but a compound, ingredient, or other derivative of it is weighed instead, the absolute error made in weighing the derivative will give a different absolute error in the calculated substance, on account of the different weights of the two things. For instance, suppose 1.0000 g of a substance to be analyzed for potassium, and that the latter is weighed in the form of K_2PtCl_6, which contains 16.12% of potassium. Now, if an error of .0002 g, or .02%, is made in weighing the K_2PtCl_6, this will produce an error of

$.02 \times .1612 =$ about .003% in the percentage of potassium.

As an example of a calculation connected with errors in weighing and the accuracy of the results, let us suppose that 5% of a constituent is to be determined in 1.0000 g of a substance, and let us find the error in weighing the original substance, and also the error in weighing the constituent as such, which will cause an error of .05% in the result. The error is one part in 100; hence the original substance must have this proportional error, or .01 g, to produce the result. The amount of the constituent corresponding to .05% of the substance analyzed is .0005 g; hence this is the required error in it.

PROBLEMS.

12. If 2.000 g of substance analyzed has an error of .001 g, what absolute effect will this error have upon the determination of a constituent amounting to 5.00% of the substance?

13. If the error in weighing .1500 g of a substance analyzed produces an error of .20% (of the substance) in the determination of 8.00% of a constituent, what was the original absolute error?

14. If .5000 g of substance are analyzed, and a constituent amounting to 43.75% is determined with an error of .00005 g in its weight, what absolute effect will this error have upon the percentage?

15. If the error in the weight of a constituent amounting to 10.00% is .0002 g, and if this caused an error of .15 in the percentage of the constituent, what was the weight of the substance analyzed?

16. If the error in the determination of potassium in a substance is .02 in the percentage of potassium, and it is due to an error of .001 g in weighing K_2PtCl_6, which contains 16.12% of potassium, what was the weight of the substance analyzed?

17. How much of a substance must be weighed to the nearest tenth of a milligram, so that the maximum error shall be one part in 10 000?

18. With what degree of accuracy must 10 g be weighed in order that the maximum error in determining 35% of a constituent may be within one part in 50 000?

19. How much substance, weighed with an error of .0001 g, must be taken, so that this error will produce an error of .01 in the percentage of a constituent amounting to 40.00%?

20. If 2.0000 g of a silver ore are taken for analysis, and the silver, amounting to 2.00%, is weighed with an error of .00005 g, to what percentage of silver in the ore does this error correspond?

21. If 5.0000 g of a silver ore give .525% of silver chloride, which contains 75.3% of silver, and if the silver chloride is weighed with an error of .0001 g, to what percentage of silver in the ore does this error correspond?

22. If 5.000 g of an ore containing 15.00% of copper are weighed within .001 g, and if the metallic copper obtained from the ore is weighed within .0001 g, what will be the maximum error in percentage of copper in the ore when both maximum errors act in the same direction, and when they operate in opposite directions?

23. If 30.0000 g of ore containing gold to the value of $100 per cubic yard are weighed to the nearest tenth of a milligram, what is the maximum error due to this weighing, expressed in cents per cubic yard?

24. If iron in 1.000 g of ore is determined with 40.0 cc of a solution, where the error in reading the volume may reach .05 ccm, with what degree of accuracy must the ore be weighed in order that the maximum error in weighing it shall be one-half of that involved in measuring the liquid?

25. If 5.000 g of a substance are weighed within 1 mg, and 30.00% of sulphur in it are weighed as $BaSO_4$, which contains 13.73% of sulphur, with an error not greater than 1 mg, what are the maximum errors in the percentage of sulphur in the substance from the two weighings?

9. Abbreviated Multiplication. The long method of multiplying approximate numbers involves useless labor on account of finding and setting down figures which have no significance. An example of long multiplication is here given, where the useless figures are at the right of the dotted line:

$$
\begin{array}{r}
.2\,3\,3\,5\,5 \\
.5\,8\,8\,5\,5 \\
\hline
1\,1\,6\,7\,7\,5 \\
1\,1\,6\,7\,7\,5 \\
1\,8\,6\,8\,4\,0 \\
1\,8\,6\,8\,4\,0 \\
1\,1\,6\,7\,7\,5\,X \\
\hline
.1\,3\,7\,4\,5\,\underline{5}\,\underline{8}\,\underline{5}\,\underline{2}\,\underline{5}
\end{array}
$$

The principles already set forth show that the sixth figure of the result has a maximum derived error of about three units, so that the result should be given as .137456. The conditions which cause an error in this multiplication may be clearly seen when it is borne in mind that there are unknown, positive or negative figures following the last figures of the numbers multiplied, and that one of these would be multiplied by 5 and would give a product in the place marked X. In this case, if the unknown figure were as great as 2, the figure of the result to the left of X would be affected to the extent of a unit.

An abbreviated multiplication of the numbers used for the long operation is shown as follows:

$$
\begin{array}{r}
.2\,3\,3\,5\,5 \\
5\,5\,8\,8\,5.\leftarrow \\
\hline
1\,1\,6\,7\,7\,5 \\
1\,8\,6\,8\,4 \\
1\,8\,6\,8 \\
1\,1\,7 \\
1\,2 \\
\hline
.1\,3\,7\,4\,5\,\underline{6}
\end{array}
$$

The operation is performed by inverting either of the numbers under the other; then using successively the figures of the inverted number for multiplication, starting with the figure directly over the one used in each case, and carrying anything that should come from multiplying the figure to the right (any fraction amounting to .5 or over being carried as 1), and placing the first figure of the result in the right-hand column. The position of the decimal-point must be determined by inspection.

The position of the inverted number used in the preceding example is the usual one for numbers of equal length (leaving initial 0's following a decimal-point out of consideration); but when both numbers, of equal length, begin with small figures, it is advisable, in careful work, to move the inverted number one place toward the right, in order that accidental errors, due to irregularities from carrying fractions of .5 or over as units and omitting smaller fractions, may not be as great as the errors derived from apparent errors of the numbers multiplied. As an example, the multiplication of 1.333 by 1.555, both of which begin with small figures, will be shown in the usual position for other cases, and in the modified position for this particular case:

1.3 3 3	1.3 3 3 0
5 5 5.1←	5 5 5.1←
1 3 3 3	1.3 3 3 0
6 6 7	6 6 6 5
6 7	6 6 7
7	6 7
2.0 7 4	2.0 7 2 9

The derived uncertainty in the product is about eight units in the fifth figure; but the first operation shows only four figures, and is consequently not as satisfactory as the second.

When numbers of unequal length are multiplied, the longer number is the more convenient one to invert, because the usual position of the first figure of the inverted number is then under the last figure of the shorter number, so that the arrangement corresponds to that for numbers of equal length.

As an example of this case, let us multiply .0324 by 6.13425:

$$
\begin{array}{r}
.0324 \\
5243\,1.6 \leftarrow \\
\hline
1944 \\
32 \\
10 \\
1 \\
\hline
.1987
\end{array}
$$

In this operation the last two figures of the longer number were not used, hence they might as well have been omitted in setting it down.

With numbers of unequal length, the placing of the inverted number one place farther to the right than usual, for the sake of accuracy, is called for when the longer number begins with a small figure; for the size of the first figure of the shorter number is of no importance in this connection, because it multiplies a much smaller proportional error. For example, let us multiply .557 by 1.1589:

.5570
9851.1←
5570
557
279
44
5
.6455

The preceding discussion has shown that for accurate work the last figure of the result of an abbreviated multiplication should possess a considerable amount of derived uncertainty. If this point is borne in mind, there will be no difficulty in the proper use of this method.

10. Abbreviated Division. Ordinary long division leads to the employment of many unnecessary figures when approximate numbers are used; hence it is advantageous to omit the useless figures by abbreviating the process. Let us start the division of .137456 by .5885:

.5885).137456(.23
11770X
19756
17655
2101Y

It is evident that the place marked X has a maximum derived error of about ten units, because the apparent error of the divisor has been multiplied by two. The whole column under X will, therefore, be uncertain

to the same extent; but this column should not be omitted, because it is essential that a decidedly uncertain last figure should be retained throughout the operation, in order that errors due to irregularities in carrying and omitting fractions may have no appreciable effect.

There would be no use in bringing down a 0, or any other figure, into the place marked Y, for the figure to the left of this place is known to be very uncertain; hence, instead of this usual method of long division, the process is shortened by cutting off a figure from the right of the divisor, by making a dot or other mark, and thus continuing the division, with due attention to carrying from the last figure cut off, as is shown below:

```
.5·8·8·5).1 3 7 4 5 6(.2 3 3 5 7
         1 1 7 7 0
         ---------
           1 9 7 5 6
           1 7 6 5 5
           ---------
             2 1 0 1
             1 7 6 6
             -------
               3 3 5
               2 9 4
               -----
                 4 1
```

In the example just given the first figure of the quotient was small, so that the multiplication by it of the divisor's apparent error did not produce a very large derived error.

An example will now be given where the first figure of the quotient is large:

$$
\begin{array}{l}
3.\dot{2}\dot{1}\dot{2})2.9\,1\,2\,4\,7(.9\,0\,6\,\underline{7} \\
\quad\underline{2\,8\,9\,0\,8X} \\
\quad\quad 2\,1\,7 \\
\quad\quad\underline{1\,9\,3} \\
\quad\quad\quad 2\,4
\end{array}
$$

In this case, the figure to the left of X has a derived maximum error of over four units (since $9\times.5=4.5$), so that its column, instead of the one under X, may be used as the last one.

When the divisor is longer than the dividend, or when both numbers are of equal length, abbreviated divisions are carried out according to the principle that has been explained by the preceding examples; that is, the operation is carried on by the regular process of long division until a decidedly uncertain place at the right is reached, and, from this point, the process is shortened. Examples are given below:

$$
\begin{array}{lll}
.1\dot{3}\dot{0}\dot{4}\dot{7})8\,7\,2\ (6\,6.8\,\underline{4} & & .9\dot{1}\dot{0}\dot{2})8.3\,6\,1\ (9.1\,8\,6 \\
\quad\underline{7\,8\,2\,8} & & \quad\underline{8\,1\,9\,1\,8} \\
\quad\quad 8\,9\,2 & & \quad\quad 1\,6\,9\,2 \\
\quad\quad\underline{7\,8\,3} & & \quad\quad\underline{9\,1\,0} \\
\quad\quad 1\,0\,9 & & \quad\quad 7\,8\,2 \\
\quad\quad\underline{1\,0\,4} & & \quad\quad\underline{7\,2\,8} \\
\quad\quad\quad 5 & & \quad\quad\quad 5\,4
\end{array}
$$

To show that the result of an abbreviated division is correct, it is often convenient to multiply the quotient

by the divisor, using shortened multiplication for the purpose. An example, where .1703 is divided by .1633, and the result is checked, will suffice to explain the process.

```
.1633).1703   (1.0429
         1633    3361.←
         ----   -----
          700   10429
          653    6257
          ---    ----
           47     313
           33      31
           --     ---
           14  .17030
```

PROBLEMS.*

26. Multiply .234567 by .0015.
27. Multiply .08331 by 4.213.
28. Multiply .1126 by 1.471.
29. Multiply 2360 by .002131.
30. Divide 28.31 by .01562.
31. Divide .7421 by .783.
32. Divide 8000 by .0134.
33. Divide 5.52 by .001352.

11. Errors in the Use of Logarithms. The inherent errors arising from the employment of logarithms in calculations depend upon the number of places to which the table used in carried out. With a three-place mantissa the maximum error is evidently .0005, and the number corresponding to this logarithm is

* Assume apparent errors where last figures are not underlined; if last figure is underlined, assume an error of five units in it.

about 10012; hence this error may produce a variation corresponding to the multiplication or division of a number by 1.0012, or about one part in 800, and, consequently, this is the maximum error involved in expressing a number by means of a three-place table. The accuracy of logarithms increases approximately tenfold for each additional place, so that the maximum errors in representing numbers by four-, five-, six-, and seven-place logarithms are about one part in 8000, 80 000, 800 000, and 8 000 000, respectively. When two logarithms are used for an operation of multiplication or division, their errors, and also the error of the logarithm obtained by the operation, may act in different directions or in the same direction. In the latter case, the resulting number may be affected by an error three times as great as that of a single logarithm; hence the maximum error possible in an operation with two numbers is three times that mentioned above, or one part in about 270, 2700, 27 000, etc., for tables of three, four, five, etc., places.

Now, in chemical work it frequently happens that calculations of percentages are carried out to the second decimal-place, where a unit in a large percentage amounts to nearly one part in 10 000, and it is to be perceived that a four-place table, used with operations upon two numbers, is liable to introduce an error of about three units in this last figure. It is evident also that a five-place table would give accurate results in the case just mentioned, while a seven-place table is

at least one hundred times more accurate than necessary for the purpose.

It should be mentioned that the accidental errors in much of the ordinary chemical work are so great that four-place logarithms are amply accurate for its interpretation; but, still, the best chemical data require a five-place table, and it is, therefore, to be recommended for general use.

12. Interpolation. With a logarithmic table of five places, the logarithm of any number having four figures, or the four-figure number nearest to any logarithm, may be found without interpolation. Consequently, when such a table is used, interpolations are necessary only when the data or results are significant to the extent of more than four figures. The maximum error involved in omitting interpolation is evidently 5 in the fifth place of the number concerned. This amounts to one part in 2000 at the beginning of the table, with the number 1000, while at the end of the table, with the number 9999, it amounts to only one part in 20 000. It is clear, therefore, that it is much more important to interpolate when a number having five figures begins with a small figure than when the first figure is a large one. The necessity for interpolation in any particular case must be decided by a consideration of all the numbers to be used in the operation. If any of these numbers possess considerably larger apparent errors than the error introduced by omitting the interpolation, this omission is allowable.

The exceedingly simple process of interpolation would not present any difficulty to students if they would take pains to understand the reason for it, instead of blindly trying to remember some set rule for the operation.

Let us take an example of an interpolation: The logarithms of 6511 and 6512 are, respectively, .81365 and .81371; what is the logarithm of the intermediate number 65116? The logarithm increases with the number, and the increase for a unit in the fourth place of the number is 6 in the last place of the logarithm; hence the increase for 6 in the fifth place of the number must be $.6 \times 6 = 3.6$, and, consequently, the required logarithm is .81369. As another example, suppose we have given the logarithm .40522, and wish to find the number corresponding to it, knowing that the logarithms .40518 and .40535 correspond to 2542 and 2543, respectively. We see that a unit in the fourth figure of the number corresponds to a difference of 17 at the end of the logarithm, and that the given logarithm is greater by 4 than the one corresponding to 2542; the fifth figure of the number is therefore $\frac{4}{17}$ of 10, or about 2, and the required number is 25422.

The tables of "proportional parts" often given with logarithmic tables simplify such calculations as those just made.

PROBLEMS.

34. The logarithm of 1256 is .09899, while that of 1257 is .09934. What is the logarithm of 12569?

35. The number corresponding to the logarithm .58035 is 3805, while the one corresponding to .58047 is 3806. What is the number corresponding to .58040?

36. Certain logarithmic factors corresponding to 18.5° and 19.0° are respectively .18761 and .18686. What is the factor for 18.85°?

37. In correcting for altitude certain tables applying to sea-level, it is directed to subtract the logarithm .00013 for an altitude of 5000 ft., and .00026 for an altitude of 10 000 ft. What logarithms should be subtracted for altitudes of 800 and 6500 ft.?

13. Operations with Logarithms. When calculations are made by the use of logarithms, the consideration of apparent errors and significant figures should not be lost sight of, nor should interpolations be made unless the accuracy of the numbers used requires it. An example will illustrate these points: Suppose the operation $.1234 \times .75275 \div 2.1378$ is to be carried out. Since the first number has a maximum apparent error of one part in about 2500, it is to be seen that interpolation will not be necessary with the other numbers, for if we add five units to the last figure of the second number and call it .7258, the change will amount to an error of only one part in 15 000, while if we change

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2.1378 to 2.138 the error will amount to only one part in ten thousand.

log. .1234,	.09132
log. .7528,	.87668
1 − log. 2.138,	.66999
Sum,	.63799

The result is .04345, where the last figure has a derived maximum error of nearly two units. The characteristics of the logarithms were not used in the example just given, because it was easy to determine the position of the decimal-point by inspection.

PROBLEMS.

(To be performed with logarithms.)

38. $107.93 \times .1204 \div 234.78$.
39. $143.38 \times .75275$.
40. $.083 \times .1362 \div 1.1378$.
41. $2.13 \div .76421$.

PART II.

CALCULATIONS RELATING TO WEIGHTS.

14. Relative Atomic Weights. Atomic weights depend for their values upon the known composition in parts by weight of chemical compounds. If the formula and the composition of any chemical compound are given, the relative values of all the elements in it can be readily found.

For example, a compound having the formula AB contains 24.725% of A and 75.275% of B. Since the formula requires equal numbers of the two kinds of atoms, the percentages represent their relative weights; hence, if we divide each one by the other, it is found that the atomic weight of B is 3.0445 times that of A, and that if the atomic weight of B is taken as 1, that of A is .32847.

As another example, let us find the relative atomic weights of oxygen and sodium from the formula of sodium carbonate, Na_2CO_3, and its composition,

Na,	43.45
C,	11.31
O,	45.24
	100.00

Since there are two atoms of sodium to three of oxygen in the formula, the relative weights of the atoms will be obtained by dividing the percentages by these numbers of atoms; hence, $43.45 \div 2 = 21.725$ and $45.24 \div 3 = 15.08$ are these relative weights, and sodium has an atomic weight $21.725 \div 15.08 = 1.4407$ times that of oxygen.

PROBLEMS.

42. A compound AB contains .7148 parts of A and .2852 parts of B. What is the atomic weight of A compared with that of B taken as 1?

43. The compound Al_2O_3 contains 53.03% of Al and 46.97% of O. What is the atomic weight of Al compared with O as unity?

44. A compound ABC_4 contains 58.85% of A, 13.73% of B, and 27.42% of C. What is the atomic weight of A when C is taken as 1?

45. A compound AB_3 contains 2.71 parts of A and 10.635 parts of B in a total of 13.345 parts. How does the larger atomic weight compare with the smaller?

46. A compound has the formula AB, and the ratio of the atomic weights A and B is 1.3498:1. What is the percentage composition?

47. A compound contains 21.88% of A, 27.84% of B, and 50.28% of C. The ratio of the atomic weights is $A:B:C = 1:1.2726:.6568$. What is the formula of the compound expressed in whole numbers?

15. Atomic Weights Compared with a Basis. The atomic weights are merely relative quantities; hence, in order to give them definite values, some basis for comparison must be chosen. At one time oxygen taken as 100 was used as a basis, but hydrogen as 1 has been very extensively employed, since this element has the lowest known atomic weight. When hydrogen is 1, oxygen is nearly 16, as may be readily calculated from the composition of water, H_2O, which contains 11.19% of hydrogen and 88.81% of oxygen. It was formerly supposed that this ratio was exactly 1:16, but it is now known that this is not the case. Since the ratio of oxygen to the other elements can be found more directly and accurately than their ratio to hydrogen, the most approved practice at present is to regard the atomic weight of oxygen as exactly 16.000, and to use this as the basis of all the other atomic weights. According to this plan, hydrogen has an atomic weight a little greater than unity (1.008).

The calculation of the atomic weights of many of the elements on the basis of oxygen as 16.000, or with any other value for oxygen, can be made very directly in many cases, since the oxides of many elements can be accurately analyzed or synthesized. For example, let us calculate the atomic weight of copper from the composition of the oxide CuO, containing 79.90% of copper and 20.10% of oxygen. By dividing 79.90 by 20.10 it is found that the atomic weight of copper is 3.975 times that of oxygen; hence if oxygen is 16.000,

copper must be 3.975 times 16.000, or 63.60. If oxygen is taken as 100, the atomic weight of copper is $3.975 \times 100 = 397.5$. If hydrogen is taken as 1.000, so that the atomic weight of oxygen becomes 15.88, the atomic weight of copper is $3.975 \times 15.88 = 63.12$.

A less direct calculation of an atomic weight will now be explained. This will involve the consideration of a formula weight, which is the sum of the atomic weights of the atoms included in a formula; for instance, the formula weight of CuO is $63.60 + 16.00 = 79.60$. The atomic weight of chlorine cannot be compared directly with that of oxygen, because the oxides of chlorine are very unstable; but it has been found that 1.00000 g of $KClO_3$ loses .39152 g of oxygen upon being ignited; hence the remainder, .60848 g, shows the relative formula weight of KCl, and by dividing .60848 by one-third of .39152 it is found that this formula weight is 4.6624 times that of oxygen, or, when oxygen is taken as 16.000, it is 74.60. Now, it has been found by analysis that the compound KCl contains 47.52% of chlorine, and therefore its formula weight, 74.60, must contain $74.60 \times .4752 = 35.45$ parts of chlorine, which is, consequently, the atomic weight.

As another example of atomic weight calculation, suppose that 1.00000 g of KCl is found to give 1.1687 g of K_2XO_4, where all the potassium in the KCl has gone into the K_2XO_4, and that it is required to calculate the atomic weight of X, knowing the atomic weights, K = 39.15, Cl = 35.45, and O = 16.000. The formula

weight of KCl is 74.60, and it is evident that twice this formula weight of KCl, or 149.20 parts of the substance, is required to give the formula weight of K_2XO_4, because the latter contains two atoms of potassium. Hence, if we multiply 149.20 by 1.1687, the formula weight of K_2XO_4, 174.37, is obtained. Now the weight of K_2+O_4 is 142.30, so that the atomic weight of X must be $174.37-142.30=32.07$.

As a final example, a case will be explained where both the substances whose weights are compared contain the element whose atomic weight is to be found: What is the atomic weight of silicon, if 2.8234 g of $SiCl_4$ give 1.0000 g of SiO_2? An algebraic solution is probably the simplest one to use here; for instance, the statement $2.8234(x+32)=x+141.80$ can be made, from which it may be found that $x=28.22$. An arithmetical solution can be made by finding that, since 70.90 parts of Cl are replaced by 16.00 parts of oxygen, the difference, 54.90 parts, corresponds to 16.00 parts of oxygen, and one part of difference corresponds to .29144 parts of oxygen; therefore the difference 1.8234 g corresponds to .53141 g of oxygen, which is the oxygen contained in the 1.0000 g of SiO_2. From the composition of the SiO_2 the atomic weight of Si is then found to be 28.22.

PROBLEMS.

48. If the atomic weights of oxygen and chlorine have the ratio 1:2.2157, what is the atomic weight of chlorine when oxygen is 16.000?

49. If the atomic weight of calcium, based on oxygen as 16.000, is 40.1, what is this atomic weight based on hydrogen as 1.000, if the ratio of oxygen to hydrogen is 16.000:1.008?

50. If CO_2 contains 27.273% of carbon, what is the atomic weight of carbon when oxygen is 16.000?

51. If CaO contains 71.48% of Ca, what is the atomic weight of calcium compared with oxygen as 100?

52. If the compound $BaSO_4$ contains Ba, 58.85%; S, 13.73%; and O, 27.42%, what are the atomic weights of barium and sulphur, based on oxygen as 16.000?

53. If 1.0000 g of the metal M gives 1.2516 g of the oxide MO, what is the atomic weight of M, based on O = 16.000?

54. If 100.000 g of $M''SO_4$ gave 63.964 g of M''S, what is the atomic weight of M'', if O = 16.000 and S = 32.06?

55. If 1.1569 g of Ag_3XO_4 yield 1.0753 g of AgCl, what is the atomic weight of X, when O = 16.000, Ag = 107.93, and Cl = 35.45?

56. If 2.55598 g of KNO_3 lost 1.36512 g of N_2O_5 upon ignition with SiO_2, what is the indicated atomic weight of K, if N = 14.04 and O = 16.000?

57. It is found that the compound $M'X'O_3$ loses 18.44% of oxygen by ignition, leaving 81.56% of pure $M'X'$ as a residue, and it is found, further, that $M'X'$ contains 40.24% of M′ and 59.76% of X′. What are the atomic weights of M′ and X′, based on oxygen as 16.000?

58. If 1.0000 g of MO gives 1.2463 g of MCl_2, what is the atomic weight of M?

16. The Calculation of Compositions from Formulas. When the formula weight of a compound is found by adding the proper atomic weights or their multiples, the composition of the compound is thus represented in terms of the parts corresponding to the formula weight, and it is then a simple matter to calculate the composition in percentages, or in other terms.

For example, the formula weight of crystallized sodium carbonate, $Na_2CO_3.10H_2O$, is found as follows:

Na_2,	46.10
C,	12.00
O_3,	48.00
$10H_2O$,	180.16
	286.26

This addition shows that 286.26 parts of the compound contain the given amounts of the three elements and water (which would be separated into hydrogen and oxygen if the separate amounts of these elements were wanted). The amounts of these constituents contained in *one* part of the substance may

be found by dividing the parts in 286.26 by 286.26. They are as follows:

Sodium,	.1610
Carbon,	.0419
Oxygen,	.1677
Water,	.6294
Total,	1.0000

When these results are multiplied by 100, the following percentages are obtained:

Sodium,	16.10%
Carbon,	4.19
Oxygen,	16.77
Water,	62.94
Total,	100.00

It seldom happens that the apparent errors of our atomic weights are so small that percentage compositions can be carried out to the third decimal-place with accuracy.

The principle involved in finding the percentage of a group of atoms is the same as that used in connection with a single atom. For example, let us calculate the percentages of H_2O, Na_2O, and P_2O_5 in the compound HNa_2PO_4. It is evident that the formula must be doubled in order to contain the groups H_2O and P_2O_5. The doubled formula $H_2Na_4P_2O_8$ is equivalent to $H_2O.2Na_2O.P_2O_5$. The calculation is made as follows:

H_2O,	18.016	÷2.8422 =	6.34
$2Na_2O$,	124.20	÷2.8422 =	43.70
P_2O_5,	142.0	÷2.8422 =	49.96
	284.22		100.00

It sometimes occurs that a calculation in terms of groups of atoms is made from a composition in terms of the elements by a very simple process which will now be explained by an example. The composition of HNa_2PO_4 in terms of the elements is as follows:

H,	1.008	÷1.4211 =	.71
Na_2,	46.10	÷1.4211 =	32.44
P,	31.0	÷1.4211 =	21.81
O_4,	64.000	÷1.4211 =	45.04
	142.11		100.00

Now, an inspection of the formula in terms of the oxides, $H_2O.2Na_2O.P_2O_5$, shows that one-eighth of the oxygen, 5.63%, goes with the hydrogen, two-eighths go with the sodium, and five-eighths go with the phosphorus; therefore, the following calculation is readily made:

H_2O,	H	.71	6.34
	O	5.63	
$2Na_2O$,	Na	32.44	43.70
	O	11.26	
P_2O_5,	P	21.81	49.96
	O	28.15	
Total,			100.00

PROBLEMS.

Note. The necessary atomic weights are to be found in the table in the appendix.

59. Find the percentage of copper in $CuSO_4.5H_2O$.

60. Find the percentage of P_2O_5 in $HNa_2PO_4.12H_2O$.

61. What is the percentage of Al_2O_3 in $AlPO_4$?

62. What is the percentage of F in SiF_4?

63. What is the percentage of water in $Na_2B_4O_7.10H_2O$?

64. Find, in the simplest way, the percentages of K_2O and I_2O_5 in KIO_3 from the composition in terms of the elements: K = 18.29%, I = 59.28%, and O = 22.43%.

17. Calculations of Various Weights from Formulas. Atomic weights and formula weights are used in calculating the weights of substances contained in or produced by other substances, when the substances have definite chemical compositions. For example, it may be asked how much sodium there is in 5.000 g of sodium chloride. By the method set forth in the foregoing section, it is found that *one* part by weight of NaCl contains .3940 parts of sodium; hence 5.000 g of NaCl contains 1.970 g of Na.

As another example, it is required to find how much Fe_2O_3 will contain 1.0000 g of Fe. Since Fe_2O_3 contains Fe_2, it is clear from the formula weights that 159.8 parts of Fe_2O_3 will contain 111.8 parts of Fe; hence 1.0000 g of Fe will be contained in 1.429 g of Fe_2O_3.

As a further example, it is required to find how much $AlCl_3$ will be produced from the aluminium in 10.000 g of Al_2O_3. Since it is evident that there is twice as much Al in the formula Al_2O_3 as in $AlCl_3$, the formula weight of Al_2O_3, 102.2 parts, will give 266.9 parts of $AlCl_3$; hence one part of Al_2O_3 will give 2.612 parts, of $AlCl_3$, and 10.000 g of Al_2O_3 will give 26.12 g of Al_2O_3.

PROBLEMS.

65. What weight of nitrogen is contained in 44.08 g of N_2O?

66. What weight of CO_2 can be prepared from 25.00 kg of $CaCO_3$?

67. What weight of AgCl can be prepared from 11.50 oz. of silver?

68. What weight of P_2O_5 will 15.50 g of phosphorus produce?

69. What weight of $Na_2CO_3.10H_2O$ will the sodium of 1000 lbs. of NaCl produce?

70. What weight of copper will produce 100 g of $CuSO_4.5H_2O$?

18. The Calculation of Factors. It is to be noticed that such problems as those given in the preceding section require a division of one formula weight or atomic weight by another, in order to find the relation of *one* part by weight of one thing to another thing. It is evident, therefore, that if this relation is to be used frequently for calculations, it is advantageous to

make the division once for all, and then to use the resulting factor, and thus obtain the required result by a single multiplication. For example, the factor by which $CaCO_3$ must be multiplied in order to give the CO_2 contained in it is $\frac{CO_2}{CaCO_3}$ or $\frac{44.00}{100.1} = .439\underline{6}$; for, evidently, if *one* part by weight of $CaCO_3$ contains $.439\underline{6}$ parts of CO_2, we have only to multiply any weight of $CaCO_3$ by this factor to find the CO_2 in it.

PROBLEMS.

71. Calculate the factor which will give the amount of lead in $PbSO_4$.

72. Find the factor to be used when the amount of $PbSO_4$ produced from PbO is required.

73. What is the factor for finding the amount of Sb in Sb_2O_4?

74. Find the factor for $Mg_2As_2O_7 \rightarrow As$.

75. By what factor must Na_2CO_3 be multiplied to give $Na_2CO_3.10H_2O$?

76. What is the factor for finding the Fe_2O_3 produced from FeO?

19. The Calculation of Weights from Single Equations. Chemical equations, if they are true ones, always represent the weights of the substances taking part in reactions, in accordance with the atomic or formula weights of these substances. Hence, if the weight of anything taking part in a reaction is given, and if the

proper equation is written, it is easy to find the weight of any other substance represented in the equation.

For example, the equation $Zn + H_2SO_4 = ZnSO_4 + H_2$ shows that 65.4 parts of zinc react with 98.07$\underline{6}$ parts of sulphuric acid to produce 161.4$\underline{6}$ parts of zinc sulphate and 2.016 parts of hydrogen, by weight, and it is required to find the amount of zinc that will produce 3.000 g of hydrogen. Since 65.4 parts of zinc produce 2.016 parts of hydrogen, *one* part of hydrogen will be produced by $\frac{65.4}{2.016} = 32.4\underline{4}$ parts of zinc, and 3.000 g of hydrogen will be produced by 97.3$\underline{2}$ g of zinc.

Any constituent of a formula can be found by a similar calculation, by using the proper atomic or formula weights. For instance, the weight of sulphur or of SO_3 which will take part in producing 1.0000 g of hydrogen, according to the equation previously considered, may be calculated just as easily as the sulphuric acid containing them. The atomic weight of sulphur is 32.06, and the formula weight of SO_3 is 80.06; hence, upon dividing these numbers by the formula weight of H_2, 2.016, it is found that 15.90$\underline{3}$ g of sulphur and 39.71 g of sulphur trioxide are the required quantities of these substances.

PROBLEMS.

77. From the equation $CaCO_3 + 2HCl = CaCl_2 + CO_2 + H_2O$, calculate the weight of $CaCO_3$ dissolved by 100 g of HCl (the actual compound).

78. From the same equation, calculate the weight of CO_2 produced by 50.0 lbs. of HCl.

79. From the same equation, calculate the amount of hydrogen taking part in the reaction when 100.0 g of $CaCl_2$ are formed.

80. From the equation $2AgNO_3 + BaCl_2.2H_2O = 2AgCl + Ba(NO_3)_2 + 2H_2O$, calculate the amount of silver chloride which 10.00 g of barium chloride, $BaCl_2.2H_2O$, would produce.

81. From the same equation, find the amount of silver taking part in the reaction when 10.000 g of barium nitrate are formed.

82. From the equation $2KClO_3 = 2KCl + 3O_2$, calculate the amount of potassium chloride formed when 5.000 oz. of oxygen are produced.

83. If, according to the same equation, 10.00 kg of oxygen are produced, how much chlorine will the potassium chloride produced contain?

84. According to the same equation, find the amount of oxygen produced when 10.0 g of chlorine are present in the $KClO_3$.

20. Calculations from Several Connected Equations. When more than one reaction is involved in an operation, it is not necessary to calculate the amount of each

successive product in cases where only the final product is required. In such cases, however, care must be taken that the original and final formula weights correspond to one another, and proper inspection, or multiplication, of the equations is required in order that correct results may be obtained.

Let us take the following series of operations and multiply the equations in such a way that the product of one equation will correspond to the amount of that substance used in the next equation: Hydrochloric acid is to be made from sodium chloride and an excess of sulphuric acid, the resulting acid is to act upon manganese dioxide to produce chlorine, the chlorine is then to set iodine free in a solution of potassium iodide, according to the equations

$$2[2NaCl + 2H_2SO_4 = 2HNaSO_4 + 2HCl],$$
$$4HCl + MnO_2 = MnCl_2 + 2H_2O + Cl_2,$$
$$Cl_2 + 2KI = 2KCl + I_2.$$

If the amount of sodium chloride which will produce 10.000 g of iodine is required, it is not necessary to calculate successively the weights of KI, Cl, MnO_2, and NaCl, although the problem might be worked out in that way; for the amount of sodium chloride can be calculated directly from the amount of iodine, since it is to be seen from the properly multiplied equations that $2I \equiv^* 4NaCl$, or consequently that $I \equiv 2NaCl$;

*This sign, ≡, signifies "equivalent to."

hence the operation required is $\frac{117.00}{126.85} \times 10.000 = 9.223\underline{5}$ g of NaCl.

It is usually unnecessary to multiply the equations as has been done above, since a simple inspection of them is sufficient, in most cases, to show the relation of the given substance to the required one. Indeed, it is often unnecessary to write out the equations in full, as long as the proper relative quantities are kept in view; so that, for example, the reactions previously used may be expressed as follows:

$$NaCl \equiv HCl,$$
$$4HCl \equiv Cl_2,$$
$$Cl_2 \equiv I_2.$$

Therefore, $I_2 \equiv Cl_2 \equiv 4HCl \equiv 4NaCl.$

PROBLEMS.

85. It is proposed to prepare a solution of $AlCl_3$, using 100.0 g of $KAl(SO_4)_2.12H_2O$, according to the equations

$$2[KAl(SO_4)_2.12H_2O] + 6NH_4OH = 2Al(OH)_3 + K_2SO_4 + 3(NH_4)_2SO_4 + 24H_2O.$$

and

$$Al(OH)_3 + 3HCl = AlCl_3 + 3H_2O.$$

What weight of 20% HCl will be required?

86. What weight of ammonium sulphate will 100 g

of metallic iron require in making ammonium ferric alum according to the following equations?

$$Fe + H_2SO_4 + 7H_2O = FeSO_4.7H_2O + H_2,$$

$$6[FeSO_4.7H_2O] + 3H_2SO_4 + 2HNO_3 = 3Fe_2(SO_4)_3 + 2NO + 46H_2O,$$

$$Fe_2(SO_4)_3 + (NH_4)_2SO_4 + 24H_2O = 2[NH_4Fe(SO_4)_2.12H_2O].$$

87. Hydrobromic acid is prepared from 10.00 g of bromine, with sufficient water and red phosphorus; barium carbonate is prepared by precipitation with ammonium carbonate from a solution of barium chloride; then the barium carbonate is dissolved in the hydrobromic acid. What weight of crystallized barium chloride, $BaCl_2.2H_2O$, should be employed?

$$Br \equiv HBr$$
$$BaCl_2 \equiv BaCO_3$$
$$BaCO_3 \equiv BaBr_2$$

88. 10.00 g of silver are dissolved in an excess of nitric acid ($Ag \equiv AgNO_3$). From the solution, silver oxide is precipitated by means of sodium hydroxide ($2AgNO_3 + 2NaOH = Ag_2O + 2NaNO_3 + H_2O$). Lithium carbonate is dissolved in HCl ($Li_2CO_3 + 2HCl = 2LiCl + CO_2 + H_2O$). The resulting solution is evaporated to dryness to get LiCl, and the latter is dissolved in water and treated with the filtered and washed Ag_2O ($2LiCl + Ag_2O + H_2O = 2AgCl + 2LiOH$). What weight of Li_2CO_3 should be employed?

89. According to the statements in the preceding

problem, what weight of lithium bromide would be obtained by neutralizing the LiOH with HBr? (Calculate this from the weight of silver.)

21. Calculation of Results of Gravimetric Analysis in the Simplest Cases. The results of gravimetric analyses are usually expressed in terms of percentages. The substance to be analyzed is weighed in the first place, then various products representing the constituents are weighed. If it happens that the product weighed is the constituent to be determined, the calculation consists simply in finding the parts in 100. For example, if 1.2341 g of a substance give .7135 g of metallic copper, the copper in *one* part of the substance is $\frac{.7135}{1.2341} = .57814$ parts, and in 100 parts it is 57.814. Again, if 1.5003 g of substance yield by analysis .2017 g of alumina, Al_2O_3, the percentage of Al_2O_3 is $\frac{20.17}{1.5003} = 13.444$.

It is to be readily perceived that the calculation of percentages may be simplified by using exactly 1.0000 g of substance for analysis, or, if required, some other weight corresponding to a number by which it is very easy to divide, such as 5.0000 g, 2.0000 g, .1000 g, etc. For example, if 1.0000 g of substance gives .5321 g of CuO, the percentage of CuO is 53.21; if 10.000 g of an ore give .01231 g of gold, the percentage of gold is .1231; if .2000 g of a substance give .0271 g. of silver, there is 13.55% of silver in it.

When the product obtained by gravimetric analysis is not the constituent itself which it is desired to calculate, the weight of the constituent must be found by means of atomic and formula weights, as described in 17, or else from factors derived from atomic and formula weights, as is explained in 18.

For example, .9832 g of a substance give .7213 g of NaCl, and the percentage of Na_2O is required. Since the formula weights of 2NaCl and Na_2O are 117.00 and 62.10, it is evident that *one* part of NaCl will give $\frac{62.10}{117.00}$ parts of Na_2O, so that .7213 g of NaCl will give $.7213 \times \frac{62.10}{117.00}$ g of Na_2O, and the percentage of Na_2O in the substance will be $72.13 \times \frac{62.10}{117.00} \div .9832 = 38.93\underline{8}$.

If the factor representing $\frac{62.10}{117.00}$, which is .53076, had been already calculated, we should multiply the weight of NaCl by this factor, then multiply by 100 and divide by the weight of the substance used, as follows:

$$.7213 \times .53076 \times 100 \div .9832 = 38.93\underline{8}.$$

It may be observed that the last figure in the result has a maximum apparent error of nearly eight units, because it is derived from the number 62.10, which has double the maximum apparent error that such a number would ordinarily have, or about one part in 6000, on account of containing two times the approximate number 23.05.

It is to be seen that such calculations as the one just given are made simpler, as has been shown in the preceding case, by the use of a weight of the substance taken for analysis which will make the division easy, such as 1.0000 g, 2.0000 g, 5.0000 g, etc.; but there is another device in the present case which simplifies the calculation still more. This consists in the use of a weight of the original substance, which bears a simple relation to the factor to be used in the calculation.

For example, suppose .5308 g of substance are taken for the determination of Na_2O, and that .3894 g of NaCl are weighed. The calculation then consists in multiplying .3894 by the factor .5308, dividing by the original weight .5308 and multiplying by 100. Since the original weight and the factor are the same numbers, multiplying by one and then dividing by the other do not change the number operated upon, so that the only necessary operation consists in multiplying .3894 by 100, which gives 38.94% of Na_2O in the substance.

If, in the last example, 5.308 g of substance had been taken, the percentage of Na_2O would be found by multiplying the weight in grams of the NaCl by 10; if 1.0616 g ($2\times.5308$ g) of substance had been taken, it would only be necessary to divide the weight of NaCl by 2 and multiply by 100, in order to obtain the percentage.

Therefore, it is evident that in any analytical operation where a single factor is involved in the calculation,

it is easy to make the calculation extremely simple by using what may be called a factor-weight of the original substance. Such factor-weights are sometimes constructed, in the shape of a single piece, for use in certain determinations, in order that the weighing may be made easier.

PROBLEMS.

90. 1.7214 g of a substance gives by analysis .6321 g of CaO. What is the percentage of CaO?

91. .1000 g of a substance gives .0731 g of metallic copper. What is the percentage of the latter?

92. .8731 g of a substance gives .0329 g of K_2SO_4. What is the percentage of potassium?

93. 1.0000 g of a substance yields .2168 g of metallic copper. What is the percentage of CuO?

94. The sulphur in a substance is to be weighed as $BaSO_4$. What weight of substance must be taken so that the weight in grams of the $BaSO_4$ multiplied by 100 will give the percentage of sulphur?

95. The factor $Mg_2P_2O_7 \rightarrow P_2$ is .2784. How much of a substance must be weighed out, so that the weight in grams of the $Mg_2P_2O_7$ multiplied by 10 will give the percentage of phosphorus?

96. The factor for calculating Ag from AgCl is .7528. What weight of substance must be taken for analysis so that the weight in grams of the AgCl, unchanged, will give the percentage of silver?

97. The potassium in a sample of $HKCO_3$ is to be weighed as K_2SO_4. What weight of substance must be taken so that the weight in grams of K_2SO_4 multiplied by 100 will give the percentage of $HKCO_3$?

98. $BaSO_4$, obtained by precipitation from a sample of dilute H_2SO_4, is to be weighed. What weight of the dilute H_2SO_4 must be taken, so that the weight of the $BaSO_4$ in grams, multiplied by 100, will give the percentage of $HNaCO_3$ necessary to neutralize the dilute sulphuric acid?

99. What weight of a substance containing copper must be taken for analysis, so that the weight in grams of metallic copper multiplied by 10 will give the percentage of sulphur which will form Cu_2S with the copper?

100. 1.0615 g of substance are used for analysis where NaCl is weighed and Na_2O is calculated. By what number must the weight in grams of NaCl be multiplied to give the percentage of Na_2O?

22. Gravimetric Calculations, in Simpler Cases where Mixtures are Weighed. It sometimes happens, in the course of analytical work, that the calculation cannot be performed by the use of a factor and the subsequent determination of percentage. This occurs, for instance, when two elements are weighed together as analogous compounds, and then, after separation, one of them is weighed in another form. For example, suppose 1.0000 g of a substance to give .1000 g of a mixture of $SrCO_3$ and $CaCO_3$, then, after a process of separa-

tion, .0100 g of $SrSO_4$ is obtained. Let us calculate the percentages of CaO and SrO. The calculation of the percentage of SrO from the $SrSO_4$ requires no explanation here; it is .56%. But, in order to calculate the percentage of CaO, the amount of $SrCO_3$ corresponding to the $SrSO_4$ (or SrO) must be found; this must be subtracted from the weight of the mixed carbonates, in order to find the amount of $CaCO_3$; then, from the latter, the required percentage is readily found: .0100 g $SrSO_4$ corresponds to .0080 g $SrCO_3$; .0080 g of $SrCO_3$ deducted from .1000 g of $SrCO_3$ and $CaCO_3$ leaves .0920 g $CaCO_3$; and the latter corresponds to 5.16% of CaO.

PROBLEMS.

101. A substance containing potassium and sodium gives 47.21% of mixed KCl and NaCl; then the K_2PtCl_6 found amounts to 58.02% (of the substance). What are the percentages of K and Na?

102. .8312 g of substance gives .2216 g of mixed KCl and NaCl, and then .2155 g of K_2PtCl_6. What are the percentages of K_2O and Na_2O?

103. 1.0000 g of substance gives .2000 g of mixed Al_2O_3 and Fe_2O_3; then this mixture is found to contain .0500 g of Fe. What are the percentages of Fe and Al?

104. 1.0000 g of a substance gives .3000 g of a mixture of Fe_2O_3 and P_2O_5, and, after a subsequent separation, .1000 g of $Mg_2P_2O_7$ is found. What are the percentages of Fe and PO_4?

105. 2.0000 g of substance give .4000 g of a mixture of Fe_2O_3 and ZnO, and the mixture is found to contain .2000 g of Fe. What are the percentages of ZnS and FeS_2?

106. 1.0000 g of substance gives .5000 g of a mixture of $CaSO_4$ and $SrSO_4$, and, after separation, .1000 g of $CaCO_3$ is found. What are the percentages of SrO and CaO?

23. Calculation of Indirect Analyses in Simpler Cases. It happens occasionally that a substance is determined, not by weighing it or something containing it, but by weighing something else which in some way depends upon the substance to be determined for its weight. For example, the active oxygen in a higher oxide of manganese may be determined by finding the weight of carbon dioxide given off by its action upon oxalic acid in the presence of hot, dilute sulphuric acid, according to the equation

$$H_2C_2O_4 + O = H_2O + 2CO_2.$$

It is evident here that the atomic weight of the active oxygen, or 16.000 parts by weight, corresponds to twice the formula weight of CO_2, or 88.00 parts by weight, so that the factor to be used is $\frac{16.000}{88.00}$, or .18182.

As another example, the bromine or silver bromide in a mixture of AgCl and AgBr may be determined by finding the loss in weight which takes place upon heating the mixture in chlorine gas and thus changing

the AgBr to AgCl. In this operation 187.89 parts of AgBr, containing 79.96 parts of Br, will form 143.38 parts of AgCl, thus producing a loss (Br − Cl) of 44.51 parts by weight. Therefore, *one* part by weight of loss corresponds to $\frac{187.89}{44.51} = 4.221\underline{3}$ parts of AgBr, and to $\frac{79.96}{44.51} = 1.796\underline{5}$ parts of bromine.

PROBLEMS.

107. A mixture of silver chloride and silver iodide is heated in chlorine until the iodide is changed to chloride. What is the factor that must be used with the loss in weight in order to calculate the amount of iodine?

108. A mixture of lead oxide and lead sulphate is so treated that the oxide is changed to sulphate, with a consequent gain in weight. What factor must be used with this gain in weight in order to calculate the amount of lead oxide, PbO, originally present?

109. 1.0000 g of substance containing Cl and Br gives .8000 g of AgCl and AgBr together. The latter lose .1000 g upon heating in chlorine. What percentages of Cl and Br does the substance contain?

110. A mixture of KI and KCl weighing 1.0000 g was so treated that the KCl was converted into KI. The resulting, pure KI weighed 1.1000 g. What weight of KCl did the original mixture contain?

111. A mixture of $CaSO_4$ and $BaSO_4$ weighing .1000 g

was boiled with $BaCl_2$ until the $CaSO_4$ was converted into $BaSO_4$. The resulting solid, now consisting entirely of $BaSO_4$, weighed .1500 g. What weight of $CaSO_4$ did the original mixture contain?

24. Calculation of Indirect Analyses in More Complex Cases. When two compounds are weighed as a mixture, and then a common constituent is determined, or both of the compounds are converted into other compounds and the resulting mixture is weighed, the calculation of the composition of the original mixture is not quite as simple as the operations previously discussed.

For example, let us suppose 1.0000 g of a mixture of KCl and NaCl to be analyzed for Cl, the common constituent, and that .5000 g of this element are found to be present in it, and let us consider three different methods by which the amounts of KCl and NaCl can be calculated.

The first method depends upon a comparison of the amount of the common constituent found in the mixture with the amounts which the same quantity of the pure compounds would contain. If the original substance had been pure NaCl, one part of it would have contained .6060 parts of Cl; but one part of the mixture was found to contain .5000 parts of Cl, and the difference, .1060 parts, is due to the presence of KCl in it. Now, if KCl had entirely taken the place of the NaCl, one part of the substance would have contained .4752 parts of Cl, and the difference would have

been .1308 parts of Cl, so that the substance in question must contain $\frac{.1060}{.1308}$ parts of KCl in one part, or .8104 g of KCl in 1.0000 g of the mixture. The difference shows the amount of NaCl, but it may be found also by a calculation similar to the one just made.

A second method for solving the problem will now be explained. From the formula weights of Cl and KCl, 35.45 and 74.60, it is found that one part of chlorine is contained in 2.1044 parts of KCl; hence .5000 g of chlorine correspond to 1.0522 g of KCl. The excess of .0522 g in the weight of this calculated KCl over the weight of the mixture depends upon the presence of NaCl in the latter, and upon the difference, 16.10, between the formula weights of KCl, 74.60, and of NaCl, 58.50. These 16.10 parts of difference correspond to 58.50 parts of NaCl, so that *one* part of the difference corresponds to $\frac{58.50}{16.10} = 3.634$ parts by weight of NaCl; hence .0522 g of difference indicate the presence of .1897 g of NaCl in the mixture. The amount of KCl may be calculated in a similar way.

The third method to be explained for solving the problem depends upon the use of algebra. Since there is 1.0000 g of the mixture of KCl and NaCl, since the factors giving Cl from KCl and NaCl are .4752 and .6060, and since the mixed chlorides contain .5000 g of chlorine, the following simultaneous equations may

be formed, where x is the weight of KCl and y is the weight of NaCl:

$$x+y=1.0000$$
$$.4752x+.6060y=\ .5000$$

These equations give the solution, $x=.8104$ and $y=.1896$.

As another example, let us find the weight of AgI and AgBr in .3000 g of a mixture of these compounds, which upon being ignited in chlorine gives .2000 g of AgCl.

This problem may be solved according to either of the three methods used for the preceding example, for the AgCl, although not a common constituent, is a common product; but only the first of the methods referred to will be used here. If the substance, instead of being a mixture, were pure AgI, on the one hand, or pure AgBr, on the other hand, the amounts of AgCl produced would be $\frac{143.38}{234.78}\times.3000$ g and $\frac{143.38}{187.89}\times.3000$ g, or .1823$\underline{1}$ g and .2289$\underline{3}$ g, respectively. The excess of AgCl found over the amount required for pure AgI is .0167$\underline{9}$ g, while the difference that would be caused by substituting AgBr for all the AgI is .0457$\underline{2}$ g; hence the actual substitution of AgBr is $\frac{.01679}{.0457\underline{2}}\times.3000=.1102$ g AgBr. The AgI is consequently .1898 g.

PROBLEMS.

112. 1.0000 g of AgI and AgBr, mixed, gave .7000 g of AgCl upon ignition with chlorine. How much AgI and AgBr did the mixture contain?

113. 1.0000 g of a mixture of Na_2SO_4 and Li_2SO_4 is found to contain .7740 g of SO_4. What is the composition of the mixture?

114. A mixture of Na_2SO_4 and K_2SO_4 weighs .1000 g, and upon analysis gives .1527 g of $BaSO_4$. What is the composition of the mixture?

115. .3000 g of AgCl and AgBr contain .2080 g of Ag. How much Br does it contain?

116. A mixture of AgCl and AgI weighing 2.0000 g contains .7878 g of Cl and I together. What is the weight of the iodine?

117. A mixture of $CaCO_3$ and $SrCO_3$ weighing .2477 g, upon treatment with sulphuric acid gives .3199 g of $CaSO_4$ and $SrSO_4$. What weight of $CaCO_3$ did the original mixture contain?

25. Calculation of Formulas from Exact Compositions. When the composition in parts by weight of a chemical compound is given, the relative numbers of atoms or groups of atoms in its formula can be found by dividing the actual weights by the atomic or the formula weights.

For example, let us divide the percentages of the elements in sodium carbonate by the atomic weights of those elements:

	Percentages.		Atomic Weights.		Relative Numbers of Atoms.
Sodium,	43.45	÷	23.05	=	1.885
Carbon,	11.31	÷	12.00	=	.9425
Oxygen,	45.24	÷	16.00	=	2.875

The ratio of the atoms is, consequently, shown by the formula $Na_{1.885}C_{.9425}O_{2.875}$, which may be simplified to Na_2CO_3 by dividing by the smallest number in the original ratio.

It is not necessary that the composition should be expressed in terms of the elements, for the formula may be found just as readily when groups of atoms are used. For example, the composition of sodium carbonate may be expressed in terms of the oxides Na_2O and CO_2, and the formula may then be derived by dividing by the formula weights of the groups:

	Percentages.		Formula Weights.		Ratio.	Simplified Ratio.
Na_2O,	58.53	÷	62.10	=	.9425	1.
CO_2,	41.47	÷	44.00	=	.9425	1.

The formula, therefore, is $(Na_2O)_1(CO_2)_1$, or Na_2CO_3.

Again, the composition of this compound may be expressed in terms of Na and CO_3 as follows:

	Percentages.		Formula Weights.		Ratio.	Simplified Ratio.
Na,	43.45	÷	23.05	=	1.885	2.
CO_3,	56.55	÷	60.00	=	.9425	1.

The formula Na_2CO_3 is arrived at in this way also.

It is to be noticed that it is not necessary that the composition in parts by weight should be expressed in terms of percentages in order that a formula may be calculated, for if the composition by weight is expressed in any other terms whatever, the simplified ratio must evidently be the same.

It should be mentioned also that whole numbers are not always obtained by dividing by the smallest number of the original ratio, and that when fractions are thus obtained it is necessary to multiply by a number which will clear the ratio of these fractions. For example, let us find the formula for an oxide of iron:

	Percentages.		Atomic Weights		Ratio.	Simplified Ratio.	Final Ratio.
Fe,	72.38	÷	55.9	=	1.295	1.	3.
O,	27.62	÷	16.0	=	1.726	1.33	4.

In order to make whole numbers it has been necessary to multiply the simplified ratio by three. The formula is evidently Fe_3O_4.

PROBLEMS.

118. Find the formula for a compound containing

Magnesium,	11.98%
Chlorine,	34.86
Water,	53.16
	100.00

119. Find the formula from the following composition:

Potassium oxide, K_2O,	21.71%
Ferrous oxide, FeO,	16.55
Sulphuric anhydride, SO_3,	36.86
Water, H_2O,	24.88
	100.00

120. Find the simplest formula for a compound containing

Carbon,	77.34%
Nitrogen,	15.08
Hydrogen,	7.58
	100.00

121. 1.0000 g of a substance contains

Carbon,	.7196 g
Hydrogen,	.0672 g
Oxygen,	.2132 g

What is its formula?

26. Calculation of Formulas from Analyses in Cases not Involving Isomorphous Replacement. When chemical compounds are analyzed exact agreements with theory cannot usually be expected, because the substances are often not perfectly pure, and also because analytical methods are almost invariably more or less inexact. In the practical calculation of formulas

therefore, exact ratios cannot be expected, but, as a general rule, the ratios obtained approach a simple formula so closely that it may be chosen without hesitation. The fact that atoms and groups of atoms are known to combine usually in simple proportions is of much assistance in such calculations; but it should not be forgotten that certain classes of compounds, particularly those containing carbon and hydrogen, may have exceedingly complex formulas.

As an example of an application of the rule of simple proportions, let us suppose that an analysis of a compound of two elements, or groups of elements, gave a ratio, of 1:2.0476. In this case the formula AB_2 would be chosen, instead of $A_{21}B_{43}$, to which the ratio corresponds, unless, for instance, the compound was one containing carbon.

When the molecular weight of an analyzed compound is known, the formula must correspond to it; but this feature of the calculation will be left for subsequent consideration, while, at present, the calculation of formulas corresponding to the simplest ratios will be taken up.

Since ratios need not be determined very exactly to show simple formulas, it is sufficient, in most cases, to use approximate atomic and formula weights for obtaining ratios for this purpose, and some labor is often saved in this way. For example, if the following assumed analysis, which would be an excellent one in its accuracy, is subjected to formula calculation

with exact and with approximate weights, the correct formula is found equally well in the two cases.

	Analysis.		Exact Weights.		First Ratio.	Simplified Ratio.	Whole Numbers.
Na,	14.03	÷	23.05	=	.6087	1.969	2
SO_4,	29.69	÷	96.06	=	.3091	1.000	1
H_2O,	56.20	÷	18.016	=	3.120	10.09	10

	Analysis.		Approximate Weights.		First Ratio.	Simplified Ratio.	Whole Numbers.
Na,	14.03	÷	23.	=	.610	1.97	2
SO_4,	29.69	÷	96.	=	.309	1.00	1
H_2O,	56.20	÷	18.	=	3.12	10.1	10

It is to be observed that it is necessary to carry out ratios to only three figures for the purpose of finding simple formulas.

Even wider approximations in atomic and group weights than those just used answer the purpose satisfactorily in the calculation of simple formulas; hence such rough atomic weights as the following may be used for the purpose:

Ag 108, Al 27, Cl 35.5, Ca 40, Fe 56,
I 127, N 14, K 39, Mg 24, Si 28, etc.

When a ratio does not show at a glance what the relations are, even after the smallest member has been divided into the others, the simplified ratio may then be so multiplied as to give results that are close to whole numbers. For example, if the ratios of *A*, *B*, and *C* is

.512:.644:.887 = 1:1.26:1.73, it is to be seen that the simplified ratio multiplied by four gives 4:5.04:6.92, which shows the formula to be $A_4B_5C_7$.

There are many cases, however, where it is not best to take the smallest member of a ratio as unity. This happens when the constituent from which the smallest member is derived has not been as accurately determined as the other constituents, and particularly when it is derived from a small percentage which is liable to possess, on this account, a large proportional error. It is far preferable in such cases to choose as basis a more accurately determined constituent, or one giving a larger ratio number. For example, suppose the ratio of *A*, *B*, *C*, and *D* is 1:3.73:5.53:11.06, and assume that *A* has a considerable proportional error, while *C* has been very accurately determined. It would then be advisable to find the ratio on a basis of *C* as a whole number, and 6, the nearest one, would suggest itself for trial. Upon dividing all the members of the ratio by one-sixth of 5.53 the ratio 1.09:4.05:6.00:12.00 is obtained, and the formula is shown to be probably $AB_3C_6D_{12}$.

This point may be illustrated further by the calculation of the formula of an actual analysis, where the smallest ratio member is obtained from a very small percentage, and where the iodine determination is probably the most exact one, so that after inspection one-half of the first iodine ratio number is taken as unity.

	Analysis.			First Ratio.	Ratio when I = 2.00.	Whole Numbers.
Potassium,	16.94	÷ 39.2	=	.432	2.01	2
Hydrogen,	.20	÷ 1.	=	.20	.93	1
Iodine,	54.46	÷ 127	=	.429	2.00	2
Chlorine,	7.72	÷ 35.5	=	.217	1.01	1
Oxygen (diff.),	20.68	÷ 16.	=	1.293	5.93	6

The formula is $K_2HI_2ClO_6$, a result which would not be so readily found by taking the ratio number of hydrogen as unity.

PROBLEMS.

122. What is the simplest formula corresponding to the actual analysis given below?

Potassium,	4.75
Lead,	49.11
Bromine,	46.98
	100.84

123. What is the simplest formula corresponding to the actual analysis given below?

Potassium,	6.03
Lead,	30.73
Iodine,	57.57
Water,	5.26
	99.59

124. What is the simplest formula corresponding to the following actual analysis?

Rubidium,	34.58
Iodine,	36.00
Chlorine,	14.82
Oxygen,	13.15
Hydrogen,	.29
	98.84

125. Find the simplest formula for the actual analysis given below. (Assume the lead and iodine to be most accurately determined.)

Lead,	38.54
Iodine,	58.41
Hydrogen,	.20
Oxygen (diff.),	2.85
	100.00

126. Calculate the simplest formula from the following actual analysis:

Copper oxide, CuO,	66.38
Nitric anhydride, N_2O_5,	22.76
Water, H_2O,	11.26
	100.40

127. What formula corresponds to the following modification of an actual analysis?

Phosphoric anhydride, P_2O_5,	32.14
Manganous oxide, MnO,	63.78
Water, H_2O,	4.01
	99.93

128. What is the formula corresponding to the following analysis?

Silica, SiO_2,	32.03
Manganous oxide, MnO,	38.67
Calcium oxide, CaO,	29.46
	100.16

27. Calculation of Formulas from Analyses in Cases of Isomorphous Replacement. Chemical compounds which crystallize in the same form may, in many cases, crystallize together in any proportion. The interpretation of analyses of such isomorphous mixtures is important, particularly in connection with minerals, since these substances usually contain isomorphous replacements.

As an example of salts which crystallize isomorphously, we may take the alums

$$KAl(SO_4)_2.12H_2O,$$
$$NH_4Al(SO_4)_2.12H_2O,$$
$$KFe(SO_4)_2.12H_2O,$$
$$NH_4Fe(SO_4)_2.12H_2O.$$

If these four salts were mixed in various proportions in solution, various mixtures would be obtained upon crystallization, and if these mixtures were analyzed it would not be expected that the Al and Fe or the K and NH_4 would give simple ratios with each other, but by combining the ratios of these analogous elements, the general alum formula would be shown.

The following analysis of a mixture of the four salts is assumed, and the ratios are calculated from it:

	Analysis.	First Ratio.	Second Ratio.	Whole Numbers.	First Ratio Simplified.
Potassium, K,	$4.54 \div 39 =$	.116	.213	1	2.11
Ammonium, NH_4,	$1.74 \div 18 =$	.097			1.76
Aluminium, Al,	$4.25 \div 27 =$	.157	.212	1	2.85
Iron (ferric), Fe,	$3.06 \div 56 =$	.055			1.00
Sulphate, SO_4,	$40.64 \div 96 =$	.423	.423	2	7.69
Water, H_2O,	$45.75 \div 18 =$	2.54	2.54	12	46.2

It is to be seen that no satisfactory ratio can be obtained without adding the ratio numbers of the analogous constituents, but when this is done the formula

$$(K,NH_4)_1(Al,Fe)_1(SO_4)_2.12H_2O$$

is found.

The elements and radicals which may replace one another in isomorphous compounds are commonly those which have close chemical analogy and the same valence. For instance, the alkali-metals and ammonium,

NH_4, Li, Na, K, Rb, and Cs,

which divide themselves into two sub-groups,

Li, Na,

and

NH_4, K, Rb, Cs,

of more closely related members. It is probably possible for any alkali-metal to replace any other in certain cases of isomorphous compounds, but there are other cases where two of the metals of different sub-groups produce definite double salts; for instance, sodium-potassium tartrate. Probably any bivalent metal may replace any other of the same valence in certain cases, but there are certain groups of them which do this very readily; for example,

Ca″, Sr″, and Ba″;
Mg″, Mn″, and Fe″;
Mg″, Zn″, and Cd″;
Hg″ and Cu″;
Ni″ and Co″.

The univalent elements Ag′ and Cu′ readily replace each other, as do also the trivalent elements,

Al‴, Cr‴, and Fe‴.

Among the frequently isomorphous negative atoms and radicals are:

Cl′, Br′, and I′;
Cl′ and F′;
S″, Se″, and Te″;
Si and Ti;
PO_4'''', VO_4'''', and AsO_4'''';
SO_4'' and CrO_4'';

and there are many others.

When an analysis showing closely related elements or radicals, or those having the same valence, is under

consideration in regard to its formula, it should be noticed in the first place whether or not these elements or groups show simple relations in their ratios. If they do so, it can usually be assumed that the case is not one of isomorphous mixture; but if no simple relations can be found, it is best to add such ratio-numbers together, and thus attempt to obtain a simple expression for the formula.

It sometimes happens in the case of mineral analyses that it becomes necessary to regard quite dissimilar things as replacing one another. For instance, Na_2 may replace Ca; F′ often replaces (OH)′; $Al(OH)_2'$ has been found to replace Cu(OH)′; Al(OH)″ may replace a bivalent metal, and so on.

It should be mentioned also that minerals may contain impurities which are evidently not isomorphous substances. For instance, if quartz or an insoluble silicate is found in the analysis of a soluble compound, it is left out of consideration in calculating the formula.

PROBLEMS.

Find the formulas corresponding to the following analyses:

129. Analysis by C. H. Herty:

Lead,	Pb,	44.89
Iodine,	I,	17.61
Bromine,	Br,	32.59
Potassium,	K,	4.56
		99.65

130. Analysis by H. L. Wells:

Cæsium, Cs,	32.69
Chlorine, Cl,	3.32
Bromine, Br,	31.56
Iodine, I,	30.91
	98.48

131. Analysis by H. L. Wells:

Potassium, K,	9.24
Lead, Pb,	31.55
Bromine, Br,	39.27
Iodine, I,	14.57
Water, H_2O (diff.),	5.37
	100.00

132. Analysis by S. L. Penfield:

Phosphoric anhydride,	P_2O_5,	31.05
Alumina,	Al_2O_3,	22.19
Ferrous oxide,	FeO,	7.40
Manganous oxide,	MnO,	23.51
Lime,	CaO,	.54
Soda,	Na_2O,	.33
Water,	H_2O,	15.60
		100.62

133. Analysis by S. L. Penfield:

Phosphoric anhydride,	P_2O_5,	45.22
Ferrous oxide,	FeO,	13.01
Manganous oxide,	MnO,	32.02
Lithia,	Li_2O,	9.26
Soda,	Na_2O,	.29
Water,	H_2O,	.17
Insoluble matter,	SiO_2, etc.,	.29
		100.26

134. Analysis by S. L. Penfield.

Selenium, Se,	29.19
Sulphur, S,	.37
Mercury, Hg,	69.84
Cadmium, Cd,	.34
Insoluble,	.06
	99.80

135. Analysis by Penfield and Harper:

Phosphoric anhydride,	P_2O_5,	43.74
Beryllium oxide,	BeO,	15.51
Calcium oxide,	CaO,	33.67
Fluorine,	F,	5.27
Water,	H_2O,	3.70
		101.89
Deduct oxygen equivalent to F,		2.22
		99.67

Note. In this analysis the metals have been determined as oxides, but, since fluorine is present, fluorides must be present to some extent in place of oxides; therefore, since 2F's are equivalent to O, $\frac{16}{38} \times 5.27 = 2.22\%$ of oxygen have been deducted.

It is to be observed that no satisfactory formula will be obtained from this analysis unless the group OH is considered as replacing F.

136. Analysis by S. L. Penfield:

Sulphuric anhydride,	SO_3,	10.11
Chlorine,	Cl,	4.11
Aluminium oxide,	Al_2O_3,	6.60
Copper oxide,	CuO,	59.51
Water,	H_2O,	20.41
		100.74
Deduct oxygen equiv. to Cl		.92
		99.82

137. Analysis by H. L. Wells:

Silica,	SiO_2,	43.51
Aluminium oxide,	Al_2O_3,	16.30
Calcium oxide,	CaO,	.22
Cæsium oxide,	Cs_2O,	36.10
Potassium oxide,	K_2O,	.48
Sodium oxide,	Na_2O,	1.68
Lithium oxide,	Li_2O,	.05
Water,	H_2O,	1.50
		99.84

138. Analysis by Penfield and Pratt (staurolite):

Silicon dioxide,	SiO_2,	27.70
Aluminium oxide,	Al_2O_3,	53.22
Ferric oxide,	Fe_2O_3,	4.82
Ferrous oxide,	FeO,	9.72
Manganous oxide,	MnO,	.34
Magnesium oxide,	MgO,	2.66
Water,	H_2O,	1.97
		100.43

Note. In this mineral the water is firmly combined, and is evidently not water of crystallization; hence the hydrogen is to be treated like the other basic elements in the calculation. It is to be noticed also that the SiO_2 is slightly too high in this analysis on account of impurity, a fact which should be borne in mind in calculating the formula.

139. Analysis by S. L. Penfield:

Sulphur,	S,	16.22
Tin,	Sn,	6.94
Germanium,	Ge,	1.82
Silver,	Ag,	74.10
Impurities	(Zn and Fe),	.21
		99.29

140. Analysis by J. H. Pratt:

Silica,	SiO_2,	43.86
Aluminium oxide,	Al_2O_3,	24.96
Barium oxide,	BaO,	5.07
Strontium oxide,	SrO,	1.15
Calcium oxide,	CaO,	5.80
Magnesium oxide,	MgO,	.62
Potassium oxide,	K_2O,	3.40
Sodium oxide,	Na_2O,	1.80
Water,	H_2O,	13.35
		100.01

141. Analysis by H. W Foote:

Silica,	SiO_2,	27.22
Zinc oxide,	ZnO,	37.44
Manganese oxide,	MnO,	.50
Calcium oxide,	CaO,	26.25
Magnesium oxide,	MgO,	.07
Water,	H_2O,	8.56
Impurities	(Al_2O_3, Fe_2O_3),	.28
		100.32

28. Calculations by Use of Ratio Numbers. Certain calculations may be made very simply by the use of the numbers obtained by dividing percentages by the atomic, group, or formula weights of the substances represented by the percentages. Since the quotients

under consideration are the same as those comprised in the first ratio obtained in the process of calculating a formula from a percentage composition, they are here called ratio numbers.

Since a ratio number is found by dividing a percentage by an atomic, group, or formula weight, it is evident that percentages can be found by multiplying ratio numbers by the proper weights, and since one ratio number of a compound has a very simple relation to the others, it is to be perceived that such calculations are convenient and simple.

As an example, let us take the percentage of nitrogen in the compound $(NH_4)_2SO_4$, 21.24%, and find the ratio number of nitrogen by dividing by 14.04. This is found to be 1.513. Now, if we observe the empirical formula of the compound, $N_2H_8SO_4$, it is evident that the ratio number of hydrogen is four times that of nitrogen, that of sulphur is one-half that of nitrogen, while that of oxygen is twice that of nitrogen, on account of the relative numbers of these atoms present in the formula. From these ratio numbers the calculations of percentages are made by multiplying by the proper atomic weights, as follows:

Atom.	Multiple.	Ratio Number.		At. Wt.		Percentage
N,	(1)	1.513	×	14.04	=	21.24
H,	(4)	6.052	×	1.008	=	6.10
S,	($\frac{1}{2}$)	.7565	×	32.06	=	24.25
O,	(2)	3.026	×	16.00	=	48.41

The ratio numbers permit the calculation of any group of atoms in the compound, when it is noticed how many times, compared with nitrogen, the group occurs in the formula. This may be done for several groups as follows:

Group.	Multiple.	Ratio Number.		Group Wt.		Percentage.
NH_3,	(1)	1.513	×	17.06	=	25.81
H_2SO_4,	($\frac{1}{2}$)	.7565	×	98.08	=	74.19
NH_4,	(1)	1.513	×	18.07	=	27.34
SO_4,	($\frac{1}{2}$)	.7565	×	96.06	=	72.66
SO_3,	($\frac{1}{2}$)	.7565	×	80.06	=	60.56
$(NH_4)_2O$,	($\frac{1}{2}$)	.7565	×	52.14	=	39.44

It is to be noticed, furthermore, that if 100% is divided by the formula weight of $(NH_4)_2SO_4$, 132.20, the ratio number .7564 is obtained, and that this is the ratio number of any atom or group which is contained once in the formula. It is evident, therefore, that ratio numbers are readily found by dividing the formula weight of a compound into 100, as well as from the atomic or group weight and percentage of a constituent.

As an example of an application of the method just described, let us calculate the percentage of sodium in $Na_2CO_3.10H_2O$, the formula weight of which is 286.26. Now, $100\% \div 286.26 = .3493$, and since the formula contains Na_2, the ratio number is .6986 for

sodium, and $.6986 \times 23.05 = 16.10\%$ of sodium. In order that this method may be thoroughly understood, it should be compared with the usual method of calculating compositions from formulas described in 16, and it should be noticed that the variation really consists in multiplying by the reciprocal of the formula weight instead of dividing by this weight itself, and in doubling the reciprocal instead of the atomic weight.

Ratio numbers may be applied also very conveniently in determining formulas in certain cases where the composition of a compound is only partially known. For example, suppose that a compound containing zinc chloride, ammonium chloride, and water has been partially analyzed, and has been found to give 25.17% of zinc and 54.20% of chlorine, and that it is desired to find the formula of the compound. The ratio numbers of the known constituents are first found as follows:

	Analysis.		Atomic Weights.		Ratio Numbers.	Ratio in Whole Numbers.
Zn,	25.17%	÷	65.4	=	.385	1
Cl,	54.30	÷	35.5	=	1.53	4

It is thus shown that a part of the formula is $ZnCl_4$, and, since zinc chloride is $ZnCl_2$, there must be $2NH_4Cl$ in it, so that the formula without water is $(NH_4)_2ZnCl_4$. The next step is to calculate the percentage of NH_4, which must have twice as great a ratio number as zinc. or .770. by multiplving this number by 18, the

approximate group weight of NH_4. This gives 13.86% of NH_4, and the water is found by difference, and then its ratio number is found as follows:

	Analysis.	Formula Weight.	Ratio.	Ratio in Whole Numbers.
Zn (found),	25.17%		.385	1
Cl (found),	54.30		1.53	4
NH_4 (calculated),	13.86		.770	2
H_2O (difference),	6.67	÷ 18 =	.371	1
	100.00			

It is thus shown that the formula is $(NH_4)_2ZnCl_4.H_2O$.

Another convenient use of ratio numbers is in the calculation of compounds from the analyses of mixtures in cases where common constituents are present. For example, suppose a mixture which is known or assumed to contain NaCl, KCl, NaBr, Na_2SO_4, and $CaSO_4.2H_2O$ to give the following analysis and ratio:

	Analysis.			Ratio.
Na,	35.20%	÷ 23.05	=	1.5271
K,	1.14	÷ 39.15	=	.0291
Ca,	1.45	÷ 40.1	=	.0362
Cl,	49.65	÷ 35.45	=	1.4006
Br,	.79	÷ 79.96	=	.0099
SO_4,	10.46	÷ 96.06	=	.1089

Since the mixture is assumed to contain only one salt of K, Br, and Ca, the amounts of these may be

calculated directly from the ratio numbers, as follows:

KCl,	.0291	× 74.60	= 2.17%
$CaSO_4.2H_2O$,	.0362	× 172.2	= 6.23
NaBr,	.0099	× 103.01	= 1.02

Since an amount of SO_4 corresponding to the calcium ratio number, .0362, has been used in forming $CaSO_4.2H_2O$, there remains an amount of SO_4 corresponding to the difference between .0362 and the total ratio number of SO_4, .1089, which represents Na_2SO_4. The percentage of the latter salt is, therefore, calculated as follows:

$$.1089 - .0362 = .0727$$
$$Na_2SO_4,\ .0727 \times 142.16 = 10.33\%.$$

The percentage of NaCl may be calculated from the difference between the K and the Cl ratio numbers, as follows:

$$1.4006 - .0291 = 1.3715$$
$$NaCl,\ 1.3715 \times 58.50 = 80.23\%.$$

The percentage of NaCl may be found in another way, from the sodium ratio number by deducting from it the ratio number corresponding to NaBr and twice the ratio number corresponding to Na_2SO_4, as follows:

$$1.5271 - .0099 - (2 \times .0727) = 1.3718$$
$$NaCl,\ 1.3718 \times 58.50 = 80.25\%.$$

The close agreement of the two calculations of NaCl shows greater accuracy in the assumed analysis than would usually occur in practice, for the metallic radicals here almost precisely balance the acid radicals, while a somewhat greater discrepancy would be expected in an actual analysis from accidental errors.

The accuracy of the calculations just made may be checked by comparing the sum of the percentages of the compounds with that of the separate constituents; but to the latter must be added the undetermined water belonging to the $CaSO_4.2H_2O$, which is $.0362 \times 36.036 = 1.30\%$. The comparison is as follows:

Na,	35.20	NaCl,	80.24
K,	1.14	KCl,	2.17
Ca,	1.45	NaBr,	1.02
Cl,	49.65	Na_2SO_4,	10.33
Br,	.79	$CaSO_4.2H_2O$,	6.23
SO_4,	10.46		
H_2O (calculated),	1.30		99.99
	99.99		

The exact agreement of the sums shows that no error has probably been made in the calculation of the compounds. It may be observed, however, that the close approach to 100% in the sum is seldom reached in actual analyses.

PROBLEMS.

(To be solved by the use of ratio numbers.)

142. The compound $Mg_2P_2O_7$ contains 63.76% of P_2O_5. How much oxygen does it contain?

143. The compound $(NH_4)_2PtCl_6$ contains 7.693% of NH_3. What is its percentage of chlorine?

144. The formula weight of $Na_2SO_4.10H_2O$ is 322.32. What percentage of water does it contain?

145. The compound $HNa_2PO_4.12H_2O$ has the formula weight 358.3. How much P_2O_5 does it contain?

146. What is the simplest formula of a compound of $(NH_4)_2SO_4$, $Fe_2(SO_4)_3$, and H_2O which contains

Nitrogen,	2.91%
Ferric oxide, Fe_2O_3,	16.57%?

147. Find the simplest formula for a compound of CsCl, $CuCl_2$, and H_2O containing

Cæsium,	49.21%
Water,	4.45

148. A mixture of NaCl, KCl, K_2SO_4, and $MgSO_4.7H_2O$ contains

Na	K	Mg	SO_3	Cl
27.68%	7.37%	1.48%	7.05%	47.32%

What are the percentages of the separate salts?

29. Calculation of Formulas in Connection with Molecular Weights. In many cases the true formula of a compound is a multiple of the simplest one, a fact which is determined by the molecular weight of

the compound in cases where that constant can be found. For example, there are several compounds of carbon and hydrogen which contain about 92.3% of carbon and 7.7% of hydrogen, a composition which gives a ratio of 1:1 for the two elements. No compound CH, which would have a molecular weight 13, is known, however; but two of the compounds have molecular weights 26 and 78, and since these weights are twice and six times 13, the formulas are multiples by 2 and 6 of the simplest formula CH, and hence they are C_2H_2 and C_6H_6.

A molecular weight needs to be known only approximately in order that a decision may be made in regard to the multiple to be chosen. For example, the compound C_2H_6O requires a molecular weight 46, while the multiples by 2, 3, 4, etc., of this formula require the molecular weights 92, 138, 184, etc., so that the determination of the molecular weight within a few units of the truth will suffice to show which formula is to be chosen.

PROBLEMS.

149. The simplest formula of a compound is CN and its molecular weight is about 52. What is its true formula?

150. If a compound containing carbon, hydrogen, and oxygen gives by analysis

Carbon,	73.00%
Hydrogen,	7.50

and its molecular weight is found to be about 81, what is its formula?

151. The ratio of carbon to hydrogen is 1:2 in a compound containing only these elements. Its molecular weight is about 114. What is its formula?

152. A compound gives by analysis

Carbon,	50.6
Hydrogen,	4.4
Nitrogen,	7.6
Chlorine,	37.2
	99.8

and its molecular weight is approximately 192. What is its formula?

PART III.

CALCULATIONS RELATING TO GASES.

30. The Effects upon Gases of Pressure, Temperature, and Volume, when One of these Conditions is Constant. When the temperature of a gas does not change, the volume varies inversely with the pressure, according to Boyle's law. For example, if one cubic foot of a gas which is under a pressure of 30 inches of mercury is put under a pressure of 15 inches of mercury, its volume will then be two cubic feet; or, in another case, if 100 ccm of a gas under a pressure of 1000 g per square centimeter is compressed to 25 ccm by a change of pressure, the final pressure will be 4000 g per square centimeter.

When the pressure of a gas does not change the volume varies directly with the absolute temperature, that is, with the temperature above −273° C., according to Gay-Lussac's law. For example, 273 ccm of a gas at 0° C. (273° abs.) would expand to 293 ccm at 20° C. (293° abs.), and would contract to 253 ccm at −20° C. (253° abs.); or, again, the volume of a gas at 300° abs. would be doubled by heating to 600° abs., and would be reduced to one-half by cooling to 150° abs.

When the volume of a gas does not change, the pressure varies directly with the absolute temperature. For example, if a gas in a closed space has a pressure of one atmosphere at 280° abs., it would have a pressure of two atmospheres when heated to 560° abs.

The three features of the gas laws that have been mentioned may be expressed by proportions, where P and P', T and T', and V and V' indicate the original and final pressures, absolute temperatures, and volumes:

$$V:V' = P':P \text{ (when } T \text{ is constant);}$$
$$V:V' = T:T' \text{ (when } P \text{ is constant);}$$
$$P:P' = T:T' \text{ (when } V \text{ is constant).}$$

From these proportions formulas for calculating one unknown member when the other three are known may be derived as follows:

$$V' = V \times \frac{P}{P'} \quad \text{and} \quad P' = P \times \frac{V}{V'};$$

$$V' = V \times \frac{T'}{T} \quad \text{and} \quad T' = T \times \frac{V'}{V};$$

$$P' = P \times \frac{T'}{T} \quad \text{and} \quad T' = T \times \frac{P'}{P}.$$

It is somewhat difficult to apply these formulas mechanically to problems without making mistakes; but the matter is very simple if it is noticed that in every case the calculation is performed by multiplying a volume, temperature, or pressure by a fraction formed by two known pressures, volumes, or temperatures

$\left(\frac{P}{P''}, \frac{P'}{P}, \frac{T'}{T}, \frac{V}{V''}, \text{ or } \frac{V'}{V}\right)$; for then the fraction may be always properly made by noticing whether the result is to be greater or less than the number multiplied.

For example, the following problem is to be solved. If a gas measures 100 ccm under a pressure of 700 mm, what will be its volume under a pressure of 800 mm? It is to be seen that the increase in pressure will diminish the volume; hence $100 \times \frac{700}{800}$ will give the answer, 87.5 ccm.

As another example, the temperature is required which will change 100 ccm of a gas at 10° C. to 200 ccm. The temperature must be greater; hence 283° abs. $\times \frac{200}{100}$ gives 566° abs., the answer.

As a third example, the temperature is required which will produce a pressure of $2\frac{1}{2}$ atmospheres upon some air in a closed space which has a pressure of one atmosphere at 27° C. The temperature will be higher; hence 300° abs. $\times \frac{2\frac{1}{2}}{1} = 750°$ abs., the answer.

PROBLEMS.

153. One cubic foot of a gas is under a pressure of 20 inches of mercury. What will be its volume (with unchanged temperature) when the pressure is changed to 30 inches?

154. 100 ccm of a gas, under 760 mm pressure, is

increased to 500 ccm by change of pressure. What is this pressure at constant temperature?

155. 20 cu. ft. of air at 10 lbs. per sq. in. would occupy what volume at 15 lbs. per sq. in., the temperature remaining unchanged?

156. If the volume of a gas is 150 ccm when its pressure is 760 mm, what will be its pressure when its volume is 140 ccm (the temperature remaining unchanged)?

157. If the pressure of a gas is 700 mm when its volume is 100 ccm, what will be its volume when the pressure is 760 mm (the temperature remaining unchanged)?

158. If the volume of a gas is 123 ccm at 750 mm, what will be the pressure when the gas occupies 150 ccm (the temperature remaining unchanged)?

159. If the volume of a gas, under a pressure of 740 mm, is 121 ccm, what volume will a pressure of 760 mm give (the temperature remaining unchanged)?

160. A gas occupies 100 ccm at 0°. What will be its volume at 100° if the pressure remains unchanged?

161. A gas has a volume of 100 cu. ft. at −20° C. At what temperature will its volume be 50 cu. ft. if the pressure remains unchanged?

162. The temperature of 500 ccm of gas is 500° C. What will be its volume at −100° C. under constant pressure?

163. The temperature of 100 ccm of gas is −200° C.

At what temperature, under constant pressure, will it occupy 1000 ccm?

164. What will be the volume, under constant pressure, at 27° of a gas which has a volume of 100 ccm at −73°?

165. What will be the temperature, under constant pressure, of 100 ccm of a gas which occupies a volume of 273 ccm at 0°?

166. If a gas in a closed space of constant volume has a temperature of 100° and a pressure 700 mm, at what temperature will the pressure be 100 mm?

167. If a gas is under a pressure of 10 atmospheres at 1000°, what will be the pressure at −20°, provided that the volume is constant?

168. The temperature of some air is 50° and its pressure is 760 mm. What will be its temperature, with the same volume, at 780 mm pressure?

31. The Effects upon Gases of Pressure, Temperature, and Volume, when All of these Conditions are Variable. When the three conditions of a gas are given, and two of the conditions are then changed, the unknown third condition can be readily calculated by considering the effects of the two changes separately, according to the principles set forth in the preceding section.

For example, if 100 ccm of a gas are at 27° and 700 mm, and it is required to find the volume at −73° and 100 mm, it is to be noticed that the temperature change will decrease the volume to the extent of $\frac{200° \text{ abs.}}{300° \text{ abs.}}$,

while the pressure change will increase it in the proportion $\frac{700}{100}$, and hence the new volume will be $100 \times \frac{200}{300} \times \frac{700}{100} = 466\frac{2}{3}$ cc.

Let us take another problem: If a gas measures 100 ccm at 0° and 1000 mm, at what pressure will it measure 200 ccm at a temperature of 300°? The pressure will be decreased $\frac{100}{200}$ by the change in volume, while it will be increased $\frac{573}{273}$ by the change in temperature. Therefore the new pressure will be $1000 \times \frac{100}{200} \times \frac{573}{273} = 1049.5$ mm.

Let us take one more problem, where the temperature is the unknown quantity: 100 ccm of a gas at 27° and 700 mm will require what temperature to give 200 ccm at 500 mm? The change in volume will require a higher temperature to the extent of $\frac{200}{100}$, while the change in pressure will require a lower temperature, indicated by $\frac{500}{700}$; therefore the final temperature will be 300° abs. $\times \frac{200}{100} \times \frac{500}{700} = 428\frac{4}{7}$° abs., or $155\frac{4}{7}$° C.

A general formula may be used for the calculations under consideration. It is $\frac{P'V'}{T'} = \frac{PV}{T}$, where V, T, and P are the original conditions of a gas, and V', T', and P'

are the changed conditions of the same gas. Boyle's law requires for constant temperature that $P'V' = PV$; then, if the temperature changes, it follows from Gay-Lussac's law that the general formula given above is true. The application of this formula is an algebraic process which is liable to be used mechanically without the application of reasoning-power; hence the arithmetical method previously given appears preferable as a means of mental discipline. However, let us apply the formula to the last example explained by the other method. We have, by substitution in the formula, $\frac{500 \times 200}{T'} = \frac{700 \times 100}{300}$; hence $T' = \frac{500 \times 200 \times 300}{700 \times 100} = 428\frac{4}{7}°$ abs.

PROBLEMS.

169. Some hydrogen which occupies a volume of 100 ccm at 700 mm and 127° C. has the pressure changed to 600 mm and the temperature to 27° C. What is the resulting volume?

170. 1000 ccm of nitrogen at 0° and 760 mm would be expanded to 1500 ccm at 600 mm at what temperature?

171. 10 cu. ft. of air at 17° C. and 30 inches barometric pressure would have what pressure when the volume was 9 cu. ft. and the temperature was −13° C.?

172. 28.3 ccm of nitrogen are measured at 20.5° C. and 750 mm. What would be its volume at 0° and 760 mm?

32. The Weights of Gases under Varying Conditions. The weights of unit volumes of the different gases are

known under the standard conditions of a temperature of 0° and a barometirc pressure of 760 mm. Therefore, when the weight of a given volume of a gas under other than standard conditions is required, it is necessary to make a calculation according to the preceding section in order to find the weight.

For example, it is known that 1 ccm of nitrogen weighs .001250 g under standard conditions, and the weight of 25.0 ccm of this gas at 17° and 750 mm is wanted. We may first find the volume of the 25.0 ccm of nitrogen under standard conditions thus: $25.0 \times \frac{273}{290} \times \frac{750}{760}$ $= 23.22$ ccm. Then the weight of the gas may be found thus: $23.22 \times .001250 = .0290$ g.

From the weight of a unit volume of a gas under standard conditions, or under any other known conditions, the weight of any volume of the gas under any other conditions can be readily calculated in a similar way.

For example, if the weight of 1 ccm of air under standard conditions is .001292 g, the weight of 1000 ccm at 27° and 767.6 mm is $.001292 \times \frac{273}{300} \times \frac{767.6}{760} \times 1000$ $= 1.187$ g. In the calculation just made the fractions were chosen by noticing that the higher temperature made a given volume of the gas weigh less while the greater pressure made it weigh more.

PROBLEMS.

173. If 1 ccm of air at 0° and 760 mm weighs .001292 g, what will 1 ccm weigh at 100° and 770 mm?

174. If 1 ccm of nitrogen weighs .001250 g at 0° and 760 mm, what is the weight of 10.0 ccm at 770 mm and 17°?

175. If 21.3 ccm of a gas at 15.2° and 752.3 mm weighs .0304 g, what is the weight of 1 ccm at 0° and 760 mm?

176. If 40.35 ccm of a gas at 19.5° and 753.2 mm weigh .0713 g, what is the weight of 1 ccm at 100° and two atmospheres pressure (one atmosphere being 760 mm)?

33. Densities and Molecular Weights of Gases. The relative densities of gases and vapors are the relative weights of equal volumes under the same conditions of temperature and pressure. According to Avogadro's rule, all gases and vapors contain the same number of molecules in equal volumes under the same conditions of temperature and pressure. Upon considering the two statements just given, it will be perceived that the relative densities of gases or vapors are the same as their relative molecular weights. For example, if one gas is twice as heavy as another under the same conditions, the first must have twice the molecular weight of the second; or, if one gas or vapor has a molecular weight 3.126 times that of another the densities must bear the same relation to one another.*

* It should be mentioned that most gases and vapors vary

In order that actual molecular weights may be calculated from actual gas densities, and the reverse, some gas of known density and molecular weight must be used for comparison. Since we base our atomic weights upon oxygen as 16.000, it seems best to base our molecular weights upon this gas, one liter of which weighs 1.4285 g under standard conditions, and the molecular weight of which is 32.000. (The molecule of oxygen gas is regarded as composed of two atoms, and this is the case also with the molecules of the other commoner elementary gases, H_2, N_2, and Cl_2.)

From the molecular weight of oxygen gas, 32.000, and the weight of one liter of this gas under standard conditions, 1.4285 g, it is found that that one liter of an imaginary gas having a molecular weight 1.000 would weigh $\frac{1.4285}{32.000} = .04464$ g. Therefore if we multiply the molecular weight of any gas or vapor by .04464, the weight of one liter under standard conditions is obtained; while, if we divide the weight in grams of one liter of any gas or vapor under standard conditions by .04464, or multiply by its reciprocal 22.40, the molecular weight is obtained.

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slightly from perfect gases in their physical properties; that is, they show slight differences in their rates of expansion by heat, and they do not conform exactly to Boyle's law. Hence, equal volumes of two different gases or vapors under the same conditions usually do not contain precisely the same number of molecules, and, therefore, Avogadro's rule gives only approximate results in most cases, although its variations are not usually great enough to interfere with its practical applications.

For example, the molecular weight of the gas CO is 28; hence one liter of this gas under normal conditions weighs $28 \times .04464 = .01250$ g. One liter of a certain gas weighs .7157 g; hence its molecular weight is $.7157 \times 22.4 = 16.0$ (CH_4).

The molecular weight of hydrogen is approximately 2, and the weight of one liter of it is nearly .09 g under standard conditions. These numbers may be used for rapid, approximate calculations of densities and molecular weights from one another.

For example, the molecular weight of SO_2 is, roughly, 64, and since the molecular weight of hydrogen is 2, the SO_2 is about 32 times as heavy as hydrogen, and weighs about $32 \times .09 = 2.88$ g per liter. Again, a gas which weighs 1.52 g per liter under normal conditions is $1.52 \div .09 =$ about 17 times as heavy as hydrogen, hence its molecular weight is about 34.

Sometimes the density (specific gravity) of a gas is given in comparison with pure, dry air, which is a mixture consisting chiefly of nitrogen, oxygen, and argon, and has a density corresponding to the molecular weight 28.95. Therefore, if the specific gravity of a gas compared with air is multiplied by this number, or, roughly, by 29, its molecular weight is obtained; and, conversely, if the molecular weight is divided by this number the specific gravity compared with air is obtained.

PROBLEMS.

177. The gas A is $1\frac{1}{2}$ times as dense as the gas B. What are the relative moleuclar weights?

178. Two compounds have the molecular weights 17 and 72. What are the relative densities of their gases or vapors?

179. A gas has a specific gravity of .967 compared with air. What is its approximate molecular weight?

180. Carbon dioxide, CO_2, has the molecular weight 44. What is its approximate density compared with air?

181. A certain volume of the vapor of a compound A weighs .2316 g, while the same volume of the compound B as vapor weighs .2895 g at the same temperature and pressure. If the molecular weight of A is 80.0, what is the molecular weight of B?

182. The molecular weight of a gas is 104.4. What is the weight of 1000 ccm under standard conditions?

183. One liter of a gas weighs 1.628 g under standard conditions. What is its molecular weight?

184. Calculate roughly, on the basis of hydrogen, the molecular weight of a gas which weighs 3.17 g per liter.

185. Calculate roughly the weight of one liter of carbon dioxide, molecular weight 44.

186. What is the approximate molecular weight of a vapor 36 times as dense as hydrogen?

187. If a gas has a molecular weight of 36.46, what

is its density compared with hydrogen, which weighs .08984 g per liter?

188. What is the approximate weight at 200° and 760 mm of 1 ccm of a vapor 3.87 times as heavy as air?

189. 50 ccm of a vapor at 150° and 760 mm weigh .1421 g. What is the molecular weight of the vapor?

190. What would be the volume at 127° and 760 mm of .2000 g of a vapor whose molecular weight is 120?

191. At what pressure would .8000 g of a gas at 0° with a molecular weight of 64 occupy a volume of 100 ccm?

192. What is the molecular weight of a vapor weighing .2500 g and occupying a volume of 100 ccm at 300° and 500 mm?

193. What is the volume of 5.000 g of steam, H_2O, at 100° and 760 mm?

194. The density of a gas compared with air is .9028. What is its density compared with oxygen?

195. What volume, in each case, will 32.00 g of oxygen, O_2, 2.016 g of hydrogen, H_2, and 44.00 g of carbon dioxide, CO_2, occupy at 0° and 760 mm?

34. The Calculation of Gas Volumes by Means of Equations. In consequence of Avogadro's rule, that all gases and vapors contain the same number of molecules under the same conditions, it follows that chemical equations representing the reactions of gases show, not only the weights, but also the volumes that react, provided that the formulas used are those of the molecules.

For instance, the equation $2H_2+O_2=2H_2O$ shows that two volumes of hydrogen combine with one volume of oxygen to form two volumes of steam, if the two gases and the product are under the same conditions of temperature and pressure. The equation $2CO+O_2=2CO_2$ shows that two volumes of carbon dioxide are formed by the union of two volumes of carbon monoxide and one of oxygen.

It is to be emphasized that the molecular formulas of the gases must be used in order to show the correct volume relations. Such equations as $2H+O=H_2O$ and $CO+O=CO_2$, although they indicate the proper relative weights, do *not* give the correct relative volumes. The gases hydrogen, oxygen, nitrogen, and chlorine should always be represented by their molecular formulas, H_2, O_2, N_2, and Cl_2; for to give them atomic formulas is as false as to give CH instead of C_2H_2 as the formula for acetylene.

Suppose that it is required to find the volume of oxygen at 20° which will be sufficient to burn to CO_2 and H_2O 1000 ccm of ethyl alcohol vapor at 100°, assuming that both were under normal pressure. The equation of the reaction is $C_2H_6O+3O_2=2CO_2+3H_2O$; hence it is evident that 3000 cc of oxygen would be required if the oxygen and alcohol vapor were at the same temperature. The oxygen, however, is colder, and, therefore, less expanded than the alcohol; hence its volume will be $3000\times\frac{273+20}{273+100}=2357$ ccm.

As another example, let us find the volume of air, assumed to contain four volumes of nitrogen and one of oxygen, which must be mixed with ethyl ether vapor in order to burn it completely, assuming the air and vapor to be under the same conditions. The equation is as follows:

$$C_4H_{10}O + 6O_2 + 24N_2 = 4CO_2 + 5H_2O + 24N_2;$$

hence it appears that 30 volumes of air are required for the combustion of one volume of the vapor.

PROBLEMS.

196. What volume of nitrogen and hydrogen together would be produced by decomposing 100 ccm of ammonia, NH_3, into its elements, if all the gases were under the same conditions?

197. What volume of chlorine gas would be necessary to decompose 10 ccm of NH_3, with formation of nitrogen and hydrochloric acid gases, if both were under the same conditions?

198. What volume of air (assumed to contain four volumes of nitrogen to one of oxygen) would be required to burn 1000 ccm of marsh-gas, CH_4, with the formation of CO_2 and H_2O, when the marsh-gas and air were under the same conditions of temperature and pressure?

199. If the air and marsh-gas considered in the preceding problem were at 0° and 760 mm before com-

bustion, what would be the volume of the products after combustion at 100° and 760 mm?

200. What volume of carbon monoxide, CO, at 27° and 750 mm, would be produced by the reaction of 1000 ccm of CO_2, at 750 mm and 27°, upon red-hot carbon?

201. If 1000 ccm of hydrogen and 1000 ccm of oxygen, both at 27° and 780 mm, were mixed, and the mixture was ignited, at what temperature would the mixture resulting from the combustion have a volume of 3000 cc and a pressure of 780 mm?

202. If a mixture of 1000 ccm of air, composed of four volumes of nitrogen and one volume of oxygen, and 500 ccm of carbon monoxide, both at 0° and 760 mm, were mixed and ignited in a closed vessel of 1500 ccm capacity, what pressure, in atmospheres, would be produced by the mixture resulting after the ignition if the temperature reached 1500°?

35. The Relations of Weights and Volumes in Terms of Grams and Liters. Equations frequently represent reactions in which both gases and solids or liquids take part, and it is often required to find the volume of a gas or vapor corresponding to a certain weight of a solid or liquid taking part in the reaction, or the reverse. Such calculations may be made, of course, by finding the required volume from the calculated weight, or the required weight from the given volume, according to the methods already discussed; but a simpler method than this will now be presented.

This method depends upon the fact that *the molecular weight in grams of any gas or vapor occupies 22.40 l (22 400 ccm) under standard conditions* The number 22.40 is the reciprocal of .04464, the weight in grams of one liter, under standard conditions, of an imaginary gas having a molecular weight 1.000.* It is readily seen that 1.000 g of such a gas would occupy 22.40 l; hence the general statement given above follows from Avogadro's rule.

Therefore, the following weights of gases have a volume of 22.40 l at 0° and 760 mm:

2.016 g of H_2,
28.08 g of N_2,
32.00 g of O_2,
28.00 g of CO,
44.00 g of CO_2,
30.048 g of C_2H_6, etc.

In cases where vapors do not exist as such at 0° and 760 mm, statements can be made of the weights that would occupy 22.40 l if they did exist under those conditions. For example, if steam existed at 0° and 760 mm, 18.016 g of it would occupy 22.40 l; hence the volume of steam produced by a given weight of water at 100°, or at a higher temperature, is readily found. Again, since the molecule of mercury vapor is the single atom Hg, 200 g of it would occupy 22.40 l, or

* See 33.

1.000 g of it would occupy 112 ccm at 0° and 760 mm. if it existed under those conditions; hence at 360°, under standard pressure, 1.000 g of it would measure $112 \times \frac{273+360}{273}$, or 260 ccm.

When we have an equation which represents gas-molecules correctly, it will show the corresponding volumes of gases and weights of solids if the proper formula weights are given to the solids, and the volume 22.40 l is given to each molecule of gas.

For example, $Zn + H_2SO_4 = ZnSO_4 + H_2$ shows that 65.4 g of zinc and 98.08 g of sulphuric acid give 22.40 l of hydrogen under standard conditions; hence 1.000 g of zinc gives 342.5 ccm of hydrogen, 1.000 l of hydrogen requires 2.92 g of zinc, etc.

Let us calculate the weight of $KClO_3$ required to produce 1000 ccm of O_2 under standard conditions. The equation, with the corresponding weight and volume, is as follows:

$$2KClO_3 = 2KCl + 3O_2;$$
$$245.2 \text{ g}; \qquad 3 \times 22.40 = 67.20 \text{ l}.$$

Hence, 1.000 l of oxygen requires $\frac{245.2}{67.20} = 3.649$ g of $KClO_3$.

As an example of a somewhat more complex case, let us find the volume of air, assumed to be composed of 4 vols. N_2 and 1 vol. O_2, necessary to burn the

acetylene from 10 g of calcium carbide. The equations are as follows:

$$CaC_2 + 2H_2O = H_2C_2 + Ca(OH)_2 \text{ and}$$
$$H_2C_2 + 2\tfrac{1}{2}O_2 + 10N_2 = H_2O + 2CO_2 + 10N_2.$$

The formula weight of CaC_2 is 64.1, and the acetylene produced by this weight in grams requires $12\frac{1}{2}$ volumes of the air, or $22.40 \times 12\frac{1}{2} = 280$ liters. One gram of calcium carbide requires, therefore, 4.37 l, and 10 g require 43.7 l of air under standard conditions.

PROBLEMS.

Note. A degree of accuracy within one part in 100 will suffice for most problems of this sort, so that approximate atomic weights may be used for the problems given here.

203. According to the equation

$$Fe + H_2SO_4 = FeSO_4 + H_2,$$

what volume of hydrogen at 20° and 780 mm would be produced by 20.0 g of iron?

204. What volume of CO_2 at 0° and 760 mm would be produced by 30.0 g of $KMnO_4$ and an excess of oxalic acid, in the presence of dilute H_2SO_4, according to the equation

$$2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 = K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O?$$

205. What pressure, in terms of atmospheres of 760 mm each, would be produced by dissolving 20.0 g of

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zinc in hydrochloric acid and confining the hydrogen produced in a space of 200 ccm at 17°?

206. If 5.00 g of water are placed in a closed vessel of 1000 ccm capacity, and it is heated until the pressure amounts to 20 atmospheres of 760 mm each, what is the temperature?

207. What weight of metallic iron and of 50% sulphuric acid would be required to fill a balloon of 10 000 l capacity with hydrogen at 17° and 760 mm?

208. What weight of potassium chlorate, when put into a closed vessel of 500 ccm capacity, beyond the space occupied by the potassium chloride formed, would give a pressure of 20 atmospheres, of 760 mm each, at 500° from the pressure of the evolved oxygen?

209. What volume of oxygen, under standard conditions, would be required to burn the hydrogen produced by dissolving 20.0 g of zinc in an acid?

210. What volume of chlorine, under standard conditions, would be produced by the reaction upon MnO_2 of the hydrochloric acid set free from 100 g of NaCl by the action of H_2SO_4, according to the equations

$$NaCl + H_2SO_4 = HNaSO_4 + HCl \text{ and}$$
$$4HCl + MnO_2 = MnCl_2 + 2H_2O + Cl_2?$$

Do not determine the weight of any intermediate product.

211. To find the purity of a sample of sodium nitrate, .2500 g of it were so treated as to convert its nitrogen into the gas NO, which measured 65.5 ccm, when dry, at 20° and 760 mm. What was the percentage of

$NaNO_3$ in the substance? Do not find the weight of the gas NO, but simply the weight of $NaNO_3$ corresponding to it.

212. To find the purity of a sample of $HNaCO_3$, .2000 g of it were treated with acid, and the resulting CO_2 was found to measure 53.3 ccm, when dry, and at 17° and 750 mm. What was the percentage of $HNaCO_3$ in the substance? Do not find the weight of the CO_2.

213. What volume of air, under standard conditions, assumed to contain 4 vols. N_2 and 1 vol. O_2, would be required to oxidize 1 kg of FeS_2 to SO_2 and Fe_2O_3?

214. What volume of air, under standard conditions, assumed to contain 4 vols. N_2 and 1 vol. O_2, would be required to burn 10 g of C_2H_5OH to H_2O and CO_2?

36. The Relations of Weights and Volumes in Terms of English Weights and Measures. It has been explained in the preceding section that the molecular weight in grams of a gas or vapor occupies 22.40 liters under standard conditions. Now, since it happens that the relation between grams and liters is almost precisely the same as that between avoirdupois ounces and cubic feet (that is, 1000 g of water make practically one liter, while about 1000 ounces avoirdupois of water make a cubic foot), the number 22.4 can be made use of in calculating weights of substances and volumes of gases in terms of English weights and measures.

For example, the equation

$$2KClO_3 = 2KCl + 3O_2$$

indicates that 245.2 oz. of $KClO_3$ will produce 3×22.4 cubic feet of oxygen under standard conditions. Moreover, it is evident that ounces and cubic feet may be substituted for grams and liters in the problems of the preceding section without change in their numerical features.

When problems are presented in which temperatures are given in terms of Fahrenheit degrees, it is best to convert these into their centigrade equivalents at the outset, rather than to use a law of expansion based upon Fahrenheit degrees. To make this conversion it is only necessary to subtract 32 from the Fahrenheit temperature and multiply the remainder by $\frac{5}{9}$. For example, $70°$ F. $= (70 - 32) \times \frac{5}{9} = 21\frac{1}{9}°$ C. To make the reverse conversion it is necessary to multiply the centigrade degrees by $\frac{9}{5}$ and then add 32. For example, $15°$ C. $= (15 \times \frac{9}{5}) + 32 = 59°$ F.

When pressures are given in terms of inches of mercury, these can be used for calculations as readily as millimeters, but it is to be noticed that the standards used for normal atmospheric pressure, 760 mm and 30 inches, do not exactly agree, since 30 inches make about 762 mm and 760 mm make 29.92 inches. The difference is not important, however, since calculations in terms of atmospheres are usually only approximations, on account of the lack of precision in Boyle's law, particularly at high pressures.

PROBLEMS.

215. What volume of CO_2 at 77° F. and 30 inches pressure will be produced by the action of acid upon 10 lbs. of $CaCO_3$?

216. How many pounds of water are required to make 100 cu. ft. of steam at 212° F. and at standard pressure?

217. What is the weight in ounces of 10 cu. ft. of steam at a pressure of 2 atmospheres and a temperature of 250° F.?

218. The difference in weight, in pounds, is required between 10 000 cu. ft. of hydrogen and the same volume of air, both under standard conditions, when air is assumed to be 14.38 times as heavy as hydrogen, in order to find the lifting-power of a balloon.

219. The weight of iron is required which will produce sufficient hydrogen to fill a cylinder of 3 cu. ft. capacity at a pressure of 5 atmospheres at 15°.

220. What volume of N_2O, under standard conditions, will be produced by 1 lb. of NH_4NO_3 according to the equation $NH_4NO_3 = N_2O + 2H_2O$?

221. At what temperature Fahrenheit will 1 lb. of alcohol, C_2H_5OH, as vapor, give a pressure of 10 atmospheres in a space of 2.0 cu. ft.?

37. The Variations of Gas-volumes on Account of Water-vapor. When a dry gas is put in contact with water, so that it becomes saturated with water-vapor, it expands if the pressure remains constant, or the

pressure increases, if the volume is kept constant, to an extent which depends upon the pressure or tension of the water-vapor at the prevailing temperature. In other words, the gas and the vapor act independently of each other, so that the pressure of a moist gas is due to the sum of the pressures of the two things.

Therefore, when a moist gas has been measured at a certain temperature and pressure, it is only necessary to subtract the pressure of the water-vapor at that temperature in order to find the pressure of the gas itself.

The pressure or *tension of water-vapor* cannot be calculated in a simple manner, but it has been determined experimentally for various temperatures, and a table of tensions at temperatures at which gases are likely to be measured will be found in the appendix.

As an example of the application of the correction for water-vapor, let us calculate the volume that a gas would occupy when dry and under standard conditions if it measured 100.0 ccm when moist at 20.2° and 758.5 mm. It is found from the table that the tension of aqueous vapor at 20.2° is 17.6 mm; hence the pressure upon the gas itself is $758.5 - 17.6 = 740.9$ mm. The required volume, consequently, is $\frac{273}{293.2} \times \frac{740.9}{760} \times 100.0 = 90.7\underline{7}$ ccm.

Let us calculate also the increase in volume of 100.0 ccm of a dry gas at 25° and 760 mm, if it were saturated with moisture without changing its tem-

perature and pressure. The tension of water-vapor at 25° is 23.5 mm; hence it is evident that the gas must expand in the ratio 760:783.5 in order to keep the pressure constant, and the new volume must be $100.0 \times \frac{783.5}{760} = 103.1$ ccm.

PROBLEMS.

222. 20.4 ccm of moist nitrogen at 21.2° and 763.1 mm would occupy what volume when dry at 760 mm and 0°?

223. 50 ccm of dry hydrogen at 18.3° and 758.7 mm would occupy what volume at 0° and 760 mm when saturated with water-vapor?

224. What weight of water-vapor would be formed if a drop of water were introduced at 30° above the mercury in a barometer standing at 760 mm, when the space above the mercury amounted to 10 ccm, and each mm of the tube below 760 amounted to $\frac{1}{10}$ ccm?

(*Note.* The expansion of the mercury and of the scale are not to be considered in this problem.)

225. If a dry vessel of 1000 ccm capacity has a drop of water put into it at 0°, what weight of water will evaporate?

226. Air under 760 mm pressure, saturated to the extent of 80% with moisture at 30°, contains what percentage of water-vapor by volume?

227. If 100 000 cu m of air at any pressure were saturated with moisture at 20° C., what weight of water would be deposited upon cooling to 0°?

228. If air, when the barometer stands at 760 mm, has a temperature of 20°, and its dew-point, *i.e.*, the temperature at which it is saturated with moisture, is 15°, what percentage of moisture by volume does it contain, and what is the percentage of saturation?

229. If 1 ccm of dry air, under standard conditions, weighs .0012923 g, what would be the weight of 1 ccm of air saturated with moisture at 30° under normal pressure?

230. If 1 ccm of dry air, under standard conditions, weighs .0012923 g, what percentage of water by weight will air saturated with moisture at 20° and 760 mm contain?

38. The Correction of Barometers for Temperature.* Mercury at 0° C. is used as the standard for barometric pressures, as it varies in density at different temperatures. The scale of the barometer also changes its length with changes of temperature, but not as much as the mercury, so that it is the difference between the two expansions, or the apparent expansion, that must be taken into account in correcting barometer

* Corrections for capillarity in barometers cannot be calculated simply, but they can be determined experimentally, and the various books of reference give tables for making this correction; but the correction is supposed to be applied by the makers of the instruments to the scales of barometers needing it, so that it need not be considered further here.

The tension of mercury vapor at ordinary temperatures is so slight as to be negligible in barometer readings, but when gases are measured over mercury at rather high temperatures its effect should be taken into account.

readings to 0°. The apparent expansion of mercury with a glass scale is about .00017 parts of the column at 0° for each degree C., while with a brass scale it is about .00016 parts. Therefore $1+.00017t$ and $1+.00016t$ represent the lengths of the apparent columns as compared with the columns at 0° in the two cases. Hence it is evident that a reading at $t°$ is to be divided by $1+.00017t$ or $1+.00016t$, as the case may be, in order to correct the reading to 0°.

For example, let us correct a reading of 763.2 mm made at 20° with a glass scale: $763.2 \div 1.0034 = 760.6$.

PROBLEMS.

231. A barometer with a brass scale shows a column of 754.5 mm at 19°. What is the height of the barometer corrected to 0°?

232. The reading of a barometer with a glass scale is 29.95 inches at 60° F. What is the corrected height?

233. If the corrected height of a barometer was 760 mm, what would be the reading of a barometer with a glass scale at 25°?

234. If the error in reading a barometer may be .1 mm, and a reading of 760 mm at 20° is made, and the correction for a glass scale is made by mistake for a brass-scale correction, how will the resulting error compare with the error in reading?

PART IV.

CALCULATIONS RELATING TO VOLUMETRIC ANALYSIS.

39. Standardization of Volumetric Solutions. The solutions used in volumetric analysis must have known strengths or values. Their strengths are determined, that is, they are standardized, in several different ways, of which the following are the most important:

1. By the preparation of an exact volume of a solution from an exact weight of a chemically pure substance; for example, by dissolving 17.000 g, 10.000 g, or any other exactly known weight of pure $AgNO_3$ in a graduated 1000 ccm flask, diluting with water to the mark and mixing.

2. By comparing the solution with an accurately weighed amount of some substance with which it reacts; for example, by finding that a certain weight of pure Na_2CO_3 is exactly neutralized by a certain number of cubic centimeters of an acid that is being standardized.

3. By making a gravimetric determination of some constituent of the solution; for example, by taking a measured volume of a solution of H_2SO_4, precipitating $BaSO_4$ and weighing it.

4. By comparing the solution with another volumetric solution which reacts with it, and of which the strength is known; for example, by finding that a certain volume of a solution of KOH is neutralized by a certain volume of HCl solution which has been standardized.

By either of these methods it is found that a certain number of cubic centimeters of the solution to be standardized contains, or is equivalent to, a definite weight of some chemical compound (in the last case indirectly however). For instance, in the cases suggested as examples, it might be known that:

(1) 1000 ccm $AgNO_3$ solution $\equiv$ 17.000 g $AgNO_3$;
(2) 41.3 ccm HCl solution $\equiv$ .5320 g Na_2CO_3;
(3) 50.0 ccm H_2SO_4 solution $\equiv$ .8625 g $BaSO_4$;
(4) 40.0 ccm KOH solution $\equiv$ 42.5 ccm HCl solution.

From these statements the strength of 1 ccm of each of the solutions can be found by simple division, as follows:

(1) 1 ccm $AgNO_3$ solution $\equiv$ .01700 g $AgNO_3$;
(2) 1 ccm HCl solution $\equiv$ .01288 g Na_2CO_3;
(3) 1 ccm H_2SO_4 solution $\equiv$ .01725 g $BaSO_4$;
(4) 1 ccm KOH solution $\equiv$ 1.0625 ccm HCl solution.

In the last case (4) we have not yet a definite standard; but, assuming that the HCl solution is the one mentioned in (2), it is to be found, since 1 ccm of HCl $\equiv$ .01288 g of Na_2CO_3, that 1.0625 ccm of HCl $\equiv$ .01369 g Na_2CO_3. Hence, 1 ccm of KOH will neutralize as much

HCl as .01369 g of Na_2CO_3, and the standard may be expressed thus:

(4) 1 ccm KOH = .01369 g Na_2CO_3.

When the strength or standard of a volumetric solution has been found in terms of the weight of any chemical element or compound, other chemical compounds or elements equivalent to it, or contained in it, can be found by means of formula weights, according to the methods set forth in Part II of this book. For example, continuing with the previously assumed solutions, it is found that:

(1) 1 ccm $AgNO_3$ sol. $\equiv \frac{107.93}{169.97} \times .01700 = .01079$ g of Ag;

(2) 1 ccm HCl sol. $\equiv \frac{2 \times 36.46}{106.10} \times .01288 = .008852$ g HCl;

(3) 1 ccm H_2SO_4 sol. $\equiv \frac{2 \times 1.008}{233.46} \times .01725 = .0001490$ g acid hydrogen;

(4) 1 ccm KOH sol. $\equiv \frac{98.08}{106.10} \times .01369 = .01343$ g H_2SO_4.

And, in the same way, the weight of any chemical compound or element that would be contained in 1 ccm of any of these solutions, or that would react with 1 ccm according to any equation, could be found.

As an example of a calculation from a somewhat complex equation, let us suppose that 1 ccm of a solution of $KMnO_4$, standardized with metallic iron, was

found to be equivalent to .01385 g of Fe according to the equation

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 = K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2O,$$

and let us calculate the amount of $KMnO_4$ contained in 1 ccm of the solution. The equation shows that $2KMnO_4 \equiv 10Fe$; hence $\frac{316.3}{559} \times .01385 = .007837$ g $KMnO_4$.

As another example, where two equations are involved, let us assume that the solution of $KMnO_4$, just considered, is to be used for the determination of manganese according to the equation

$$2KMnO_4 + 3MnSO_4 + 2H_2O = 5MnO_2 + K_2SO_4 + 2H_2SO_4,$$

and let us calculate, from the original standardization in terms of metallic iron, the weight of manganese equivalent to 1 ccm of the $KMnO_4$ solution. Upon inspecting the two equations it is to be seen that in the first $2KMnO_4 \equiv 10Fe$, while in the second $2KMnO_4 \equiv 3Mn$; hence $10Fe \equiv 3Mn$, and therefore $\frac{165}{559} \times .01385 = .004088$ g Mn.

A caution in regard to the avoidance of unnecessary work in such problems as the last one may be introduced here. In this calculation, for instance, it is not necessary to calculate the weight of $KMnO_4$ when only the weight of manganese is required.

PROBLEMS.

235. If 10.000 g of $AgNO_3$ are dissolved so as to make 1000 ccm of solution, how much silver will 1 ccm contain?

236. It is found that .2000 g of Na_2CO_3 are neutralized by 40.0 ccm of a solution of H_2SO_4. What weight of acid hydrogen does 1 ccm contain?

237. If 50.0 ccm of a solution of $Ba(OH)_2$ give by gravimetric analysis 1.0000 g of $BaSO_4$, what weight of real HCl will 1 ccm neutralize?

238. If 1 ccm of HCl solution will neutralize .01000 g of $CaCO_3$, and 1 ccm of the same HCl solution will neutralize 1.500 cc of a solution of NaOH, what weight of NaOH does the latter contain?

239. What weight of $AgNO_3$ must be taken to make 1000 ccm of a solution, 1 ccm of which will react with .001000 g of sodium chloride to form AgCl?

240. What weight of pure Na_2CO_3 must be taken to make 1000 ccm of a solution, 1 ccm of which will neutralize .005000 g of actual HNO_3?

241. If 1 ccm of an acid solution will neutralize .05305 g of Na_2CO_3, what weight of NH_3 will 1 ccm neutralize?

242. If 1 ccm of acid contains .0981 g of H_2SO_4, what weight of NaOH will 1 ccm neutralize?

243. If 1 ccm of $KMnO_4$ solution is equivalent to .01300 g of Fe, according to the equation

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 = K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2O,$$

with what weight of $H_2C_2O_4.2H_2O$ will 1 ccm react, according to the equation

$$2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 = K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O?$$

244. If 1 ccm of $K_2Cr_2O_7$ solution is equivalent to .01000 g of Fe_2O_3, according to the equations

$$Fe_2O_3 + 6HCl = 2FeCl_3 + 3H_2O,$$
$$2FeCl_3 + SnCl_2 = 2FeCl_2 + SnCl_4, \text{ and}$$
$$6FeCl_2 + K_2Cr_2O_7 + 14HCl = 6FeCl_3 + 2KCl + 2CrCl_3 + 7H_2O,$$

what weight of $K_2Cr_2O_7$ does 1 ccm contain?

245. If 1 ccm of $Na_2S_2O_3$ solution is equivalent to .002000 g of KIO_3, according to the equations

$$KIO_3 + 5KI + 3H_2SO_4 = 3K_2SO_4 + 6I + 3H_2O$$
and
$$2Na_2S_2O_3 + 2I = Na_2S_4O_6 + 2NaI,$$

to what weight of iodine is 1 ccm equivalent?

40. Determinations with Single Solutions. When a solution has been standardized in terms of any element or compound, it is usually convenient to find, in the first place, its equivalent in terms of the weight of the substance to be determined. After that, the calculation of the weight of this substance consists simply in multiplying the volume of the solution used by this factor.

For example, suppose that 1 ccm of an acid is equivalent to .01084 g of Na_2CO_3, and that it is desired to

determine KOH with this solution. The weight of KOH equivalent to 1 ccm of the acid is calculated as follows: $\frac{2\times 56.16}{106.10}\times .01084 = .01148$ g of KOH. Then, supposing that 30.25 ccm of the acid are used in an analysis, the total weight of KOH is evidently $30.25\times .01148 = .3473$ g of KOH.

PROBLEMS.

246. If 1 ccm of $H_2SO_4 \equiv .01520$ g of $BaSO_4$, to what weight of NH_3 will 40.3 ccm correspond?

247. If 1 ccm of $KMnO_4 \equiv .01400$ g of Fe, to what weight of Fe_2O_3 will 30.35 ccm correspond?

248. If 1 ccm of $AgNO_3$ solution contains .01000 g of Ag, to what weight of $BaCl_2.2H_2O$ will 20.1 ccm correspond, if AgCl is produced?

249. If 1 ccm of $AgNO_3$ solution will react with .005000 g of NaCl, to what percentage of chlorine in 2.0000 g of substance do 33.45 ccm correspond?

250. If 1 ccm of $K_2Cr_2O_7$ solution contains .01000 g of $K_2Cr_2O_7$, to what percentage of iron in 1.0000 g of substance do 40.00 ccm of the solution correspond, according to the equation

$$6FeCl_2 + K_2Cr_2O_7 + 14HCl = 6FeCl_3 + 2KCl + 2CrCl_3 + 7H_2O?$$

251. If 1 ccm of $KMnO_4$ is equivalent to .01431 g of Fe, according to the equation

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 = 2MnSO_4 + K_2SO_4 + 5Fe_2(SO_4)_3 + 8H_2O,$$

to what percentage of CaO in 1.000 g of substance will 10.0 ccm of the $KMnO_4$ solution correspond, according to the equation

$$5CaC_2O_4+2KMnO_4+8H_2SO_4 = 5CaSO_4+K_2SO_4+2MnSO_4+10CO_2+8H_2O?$$

41. Determinations with Two Solutions. In many methods of volumetric analysis two solutions which react with each other are employed. One of these is used in excess to produce a desired reaction, and the excess is determined by means of the second solution, so that the amount of solution used in producing the reaction may be known. In order to do this it is necessary to know the *relative strengths* of the two solutions. If they are exactly equivalent in volume to each other, *i.e.* 1 ccm of $A \equiv 1$ ccm of B, the calculation is made simply by subtracting one volume from the other, but if they are of unequal strengths, the volume of one must be converted into the equivalent volume of the other before the subtraction is performed.

For example, let us assume that 40.0 ccm of a solution of HCl are exactly neutralized by 50.0 ccm of a solution of NaOH. It is then evident that 1 ccm of NaOH sol. $\equiv$ .800 ccm of HCl sol. Now, if 30 ccm of this acid are used in neutralizing an alkaline substance and in adding an excess of the acid, and if 20.0 ccm of the NaOH solution are then required to neutralize the excess, it is found that the NaOH solution corresponds to 20.0 $\times$.800 = 16.0 ccm of the acid; hence the acid used up

by the alkaline substance amounts to $30.0 - 16.0 = 14.0$ ccm.

In the case just considered we may find, if we wish to do so, the amount of NaOH solution equivalent to the alkaline substance: Thus, 30 ccm of HCl sol. $\equiv$ $30.0 \div .800$, or 37.5 ccm of NaOH sol., and $37.5 - 20.0 =$ 17.5 ccm of NaOH solution, which is the volume corresponding to the alkaline substance neutralized.

When the relative strengths of two solutions are known, and the absolute standard of one of them is known also, it is a simple matter to find the absolute standard of the other. For example, if 1 ccm of HCl sol. $\equiv 1.250$ ccm of NaOH sol., and 1 ccm of HCl sol. $\equiv .2500$ g of Na_2CO_3, it is evident, since things which are equivalent to the same thing must be equivalent to each other, that 1.250 ccm of NaOH sol. $\equiv .2500$ g of Na_2CO_3, and that 1 ccm of NaOH sol. $\equiv .2000$ g of Na_2CO_3.

To avoid mistakes in calculating the absolute strength of one solution from that of another, it is well to observe which is the stronger of the two solutions and then to be sure that the calculated absolute strength corresponds with this observation; and it is to be noticed in this connection that the stronger solution is used in smaller volume than the weaker one when the two solutions are compared. For instance, in the preceding example 1 ccm of HCl sol. $\equiv 1.250$ ccm of NaOH sol., hence the NaOH solution is the weaker, because it takes more of it. Then, from the statements

1 ccm HCl sol. $\equiv$ .2500 g Na_2CO_3, and 1 ccm NaOH sol. $\equiv$ .2000 g Na_2CO_3, it is to be perceived that the strengths vary in the right direction.

As an example of a calculation of the results of an analysis made with two solutions, let us suppose that ammonia, NH_3, is being determined by the use of an excess of an acid solution and by finding the excess of acid by means of an alkaline solution. Let us suppose, further, that the relative strengths of the solutions are

$$1 \text{ ccm acid} \equiv 1.032 \text{ ccm alkali},$$

and that the absolute standard is

$$1 \text{ ccm acid} \equiv .05162 \text{ g } CaCO_3;$$

then let us calculate the amount of ammonia indicated if 50.0 ccm of acid and 18.5 ccm of alkali are employed. Since $CaCO_3 \equiv 2NH_3$, we have 1 ccm acid $\equiv \frac{2 \times 17.06}{100.1}$ $\times$ .05162, or .01760 g NH_3. The acid is the stronger solution; hence 18.5 ccm alkali $\equiv 18.5 \div 1.032$, or 17.9 ccm, and, therefore, $50.0 - 17.9$, or 32.1 ccm, is the volume of acid neutralized by the ammonia, and the weight of the ammonia is $32.1 \times .01760 = .565$ g.

PROBLEMS.

252. If 1 ccm of $KMnO_4$ sol. $\equiv$ 1.207 ccm of $H_2C_2O_4$ sol.; if then 40.0 ccm of the $H_2C_2O_4$ sol. are partly used up by an oxidizing substance, and 20.75 ccm. of the $KMnO_4$ are required to oxidize the excess of $H_2C_2O_4$,

to what volume of $KMnO_4$ solution does the oxidizing substance correspond?

253. 1 ccm of acid solution $\equiv$ .8315 ccm of an alkaline solution: If 50.0 ccm of the acid are partly neutralized by ammonia, and 23.15 ccm of the alkaline solution are required to neutralize the excess of acid, to what volume of the acid solution does the ammonia correspond?

254. If 1 ccm of $KMnO_4 \equiv 1.326$ ccm of $H_2C_2O_4$ solution, and 1 ccm $H_2C_2O_4 \equiv .02134$ g $H_2C_2O_4.2H_2O$, to what weight of iron is 1 ccm of the $KMnO_4$ equivalent? The equations are as follows:

$$2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 = K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$$

and

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 = K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2O.$$

255. If 1 ccm of a solution of $Ba(OH)_2$ will neutralize 2.471 ccm of a solution of $H_2C_2O_4$, and if 1 ccm of the $Ba(OH)_2$ solution will produce .05000 g of $BaSO_4$ by gravimetric analysis, what weight of NaOH will 1 ccm of the $H_2C_2O_4$ solution neutralize?

256. If 1 ccm of $Na_2S_2O_3$ sol. $\equiv$ .7321 ccm of I sol., according to the equation

$$Na_2S_2O_3 + 2I = 2NaI + Na_2S_4O_6,$$

and if 1 ccm of $Na_2S_2O_3$ sol. $\equiv$ .002310 g of KIO_3, according to the equations

$$KIO_3 + 5KI + 3H_2SO_4 = 6I + 3K_2SO_4 + 3H_2O$$

and

$$Na_2S_2O_3 + 2I = 2NaI + Na_2S_4O_6,$$

to what weight of As_2O_3 would 1 ccm of the I solution be equivalent, according to the equation

$$As_2O_3 + 4I + 10HNaCO_3 = 2Na_3AsO_4 + 10CO_2 + 5H_2O + 4NaI?$$

257. If 40.0 ccm of HCl solution gave by gravimetric determination .5735 g of AgCl, and if 1 ccm NH_4OH sol. is equivalent to .6321 ccm of the HCl sol., to what weight of Na_2CO_3 is 1 ccm of the NH_4OH solution equivalent?

258. If 1 ccm of HCl ≡ .01071 g of Na_2CO_3, and 1 ccm of NH_4OH ≡ 1.171 ccm of the HCl sol., then if some $HNaCO_3$ is treated with 50.0 ccm of the acid, and 20.15 ccm of the NH_4OH are required to neutralize the solution, what weight of $HNaCO_3$ was present?

259. If 1 ccm $H_2C_2O_4$ sol. will neutralize .8030 ccm of $Ba(OH)_2$ solution, and 1 ccm of the acid will neutralize .005305 g of Na_2CO_3; if then 30.0 ccm of the $Ba(OH)_2$ solution are treated with some CO_2 gas, and the resulting precipitate of $BaCO_3$ is removed, and 15.25 ccm of the $H_2C_2O_4$ solution are required to neutralize the liquid, what weight of CO_2 was present?

260. 40.00 ccm H_2SO_4 ≡ 39.05 ccm NH_4OH: To standardize these solutions, .3031g of Na_2CO_3 were acidified with 50.00 ccm of the H_2SO_4 solution; then, after CO_2 had been boiled off, 8.76 ccm of the NH_4OH solution were required for neutralization. 1 ccm of the NH_4OH will neutralize what weight of $H_2C_2O_4.2H_2O$?

261. 1 ccm NH_4OH ≡ 1.088 ccm HCl; and 30.15 ccm

of HCl give, by gravimetric analysis, 1.0287 g of AgCl: To determine nitrogen in 1.0000 g of a substance, the nitrogen was converted into NH_3, the latter was passed into 50.00 ccm of the HCl, and 30.00 ccm of the NH_4OH solution were required to neutralize the excess of HCl. What percentage of nitrogen did the substance contain?

262. 1 ccm of HCl $\equiv$ .005005 g of $CaCO_3$; and 1 ccm of HCl $\equiv$ .7500 ccm of NH_4OH: To determine Na_2CO_3 in .5305 g of a substance, it was acidified with 50.00 ccm of the HCl, and after boiling off CO_2, 15.85 ccm of NH_4OH were required to neutralize the liquid. What was the percentage of Na_2CO_3 in the substance?

42. Simple-factor Solutions. When a volumetric solution is used for the determination of a single substance, the calculations of results are most easily made when the factor representing the strength of the solution in terms of the substance determined is a very simple number. For instance, if 1 ccm of a $KMnO_4$ solution is equivalent to .01000 g of Fe, the calculation of the amount of iron corresponding to any given volume of the solution is exceedingly simple. Again, if 1 ccm of a solution is equivalent to .02000 g of the substance determined, or to .03000 g of it, the calculation is still very easy. Such solutions are frequently employed in practical work, and they may be called simple-factor solutions. Such solutions lose their simplicity when they are used for other determinations, so that the simple-factor permanganate solution just

considered, for example, would not be such a solution when FeO, Mn, $H_2C_2O_4$, or anything else than metallic iron was determined by means of it.

PROBLEMS.

263. If 1 ccm of a $KMnO_4$ solution is equivalent to .01000 g Fe, to how much Fe will 24.35 ccm correspond?

264. If 1 ccm of a $K_2Cr_2O_7$ solution is equivalent to .02000 g of Fe, to what weight of Fe will 40.3 ccm be equivalent?

265. If 1.0000 g of an iron ore is taken for analysis and 40.1 ccm of a $KMnO_4$ solution, 1 ccm of which corresponds to .01500 g of Fe, are required by it, what percentage of iron does the ore contain?

266. A $KMnO_4$ solution, 1 ccm of which $\equiv$ .01000 g Fe, is used for determining FeO in a substance, 1.0000 g of the substance is taken, and 40.0 ccm of the $KMnO_4$ solution are used. What is the percentage of FeO in the substance?

267. According to the equation

$$K_2Cr_2O_7 + 6FeCl_2 + 14HCl = 2KCl + 2CrCl_3 + 6FeCl_3 + 7H_2O,$$

what weight of $K_2Cr_2O_7$ must be taken to make 1000 ccm of a solution 1 ccm of which will correspond to .02000 g of Fe?

268. What weight of Na_2CO_3 must 1 ccm of a solution of HCl neutralize in order that it may correspond to .03000 g of NaOH?

43. Normal Solutions. A normal volumetric solution is one which contains as many grams of a substance per liter as will contain, react with, or be equivalent to 1.008 g of hydrogen, usually the hydrogen of acids or hydroxides. For instance, the following amounts of acids and bases contain 1.008 g of this kind of hydrogen and will make 1000 ccm of normal solutions:

The formula weight of HCl, 36.46 g
One-half of the formula weight of H_2SO_4, 49.04 g
The formula weight of KOH, 56.16 g
One-half of the formula weight of $Ba(OH)_2.8H_2O$, 157.8 g

The following weights of several salts are equivalent to 1.008 g of the hydrogen of acids or hydroxides, because they would react with it, or be produced by its reaction, and they will make 1000 ccm of normal solution:

The formula weight of $AgNO_3$, 169.97 g
One-third of the formula weight of $AlCl_3$, 44.48 g
One-half of the formula weight of $BaCl_2.2H_2O$, 122.17 g
One-half of the formula weight of $MgSO_4.7H_2O$, 123.27 g

In all reactions of neutralization or precipitation involving the simple exchange of radicals (ions), all normal solutions must be exactly equivalent to each other in volume; that is, 1 ccm of any normal acid solution will neutralize exactly 1 ccm of any normal solution of an alkaline substance; 1 ccm of the normal solution of any chloride will react with exactly 1 ccm of a normal solution of any silver salt, etc.

The relation of grams to the liter is the same as that

of milligrams to the cubic centimeter, so that the following statements, for example, can be made regarding normal solutions:

1 ccm of any normal solution $\equiv$ .001008 g H;

1 ccm of normal solution of HCl will contain .03646 g of HCl;

1 ccm of normal solution of any alkali will neutralize .03646 g of HCl;

1 ccm of normal solution of any acid will neutralize $\frac{.10610}{2}$ g of Na_2CO_3;

1 ccm of normal solution of any acid will neutralize $\frac{.0561}{2}$ g of CaO;

1 ccm of normal solution of any acid will neutralize $\frac{.1001}{2}$ g of $CaCO_3$;

1 ccm of normal solution of any alkali will neutralize $\frac{.09808}{2}$ g of H_2SO_4;

1 ccm of normal solution of any chloride will contain .03545 g of Cl;

1 ccm of normal solution of any chloride will react with .10793 g of Ag;

1 ccm of normal solution of any barium salt will contain $\frac{.1374}{2}$ g of Ba;

1 ccm of normal solution of any aluminium salt will contain $\frac{.0271}{3}$ g of Al.

Normal solutions of oxidizing and reducing substances contain in one liter as many grams of these substances as correspond to 1.008 g of hydrogen. The oxidizing substances usually give up oxygen, chlorine, bromine, or iodine, while the reducing substances usually absorb these elements. Since one atom of oxygen combines with two atoms of hydrogen, and one atom of hydrogen combines with one of chlorine, bromine, or iodine, it is evident that one cubic centimeter of normal solution of any oxidizing or reducing substance is equivalent to .001008 g of hydrogen, $\frac{.016000}{2}$ g of active oxygen, .03545 of chlorine, .07996 g of bromine, and .12697 g of iodine.

In deciding upon the amounts of substances present in, or equivalent to, normal oxidizing and reducing solutions, the reactions involved must be considered, and the amounts of active oxygen, chlorine, bromine, or iodine set free or combined by the formula weights of the substances must be found. For convenience, the amount in grams of a compound contained in, or equivalent to, one liter of normal solution, or, in other terms, the amount in milligrams corresponding to one cubic centimeter, may be called the *normal quantity* of the compound; thus, the normal quantity of H_2SO_4 is $\frac{98.08}{2}$ = 49.04 g or 49.04 mg, and the latter may be expressed as .04904 g.

Let us find the normal quantities of several oxidizing and reducing compounds:

From the equation

$$SO_2 + 2Cl + 2H_2O = H_2SO_4 + 2HCl$$

it is seen that $SO_2 \equiv 2Cl$; hence the normal quantity of SO_2 is $\frac{64.06}{2}$ g or mg. The equation

$$2Na_2S_2O_3 + 2I = 2NaI + Na_2S_4O_6$$

shows that $Na_2S_2O_3.5H_2O \equiv I$; hence the normal quantity of $Na_2S_2O_3.5H_2O$ is 248.30 g or mg. From the equation

$$As_2O_3 + 4I + 10HNaCO_3 = 2Na_3AsO_4 + 4NaI + 10CO_2 + 5H_2O$$

it is evident that $As_2O_3 \equiv 4I$, and that $As \equiv 2I$; hence, for this reaction, the normal quantity of As_2O_3 is $\frac{198.0}{4}$ g or mg, while the normal quantity of As is $\frac{75}{2}$ g or mg. It is to be seen from the equation

$$KIO_3 + 5KI + 3H_2SO_4 = 3K_2SO_4 + 6I + 3H_2O$$

that $KIO_3 \equiv 6I$, and that one-sixth of the formula weight of KIO_3 is its normal amount for this reaction. The equation

$$MnO_2 + 4HCl = MnCl_2 + 2Cl + 2H_2O$$

shows that the normal quantity of MnO_2 is one-half of its formula weight. Either of the following equations, showing the change of a ferrous compound into

a ferric one, indicates that the atomic weight of iron, the formula weight of $FeSO_4.7H_2O$, one-half of the formula weight of Fe_2O_3, etc., are the normal quantities:

$$2FeO + O = Fe_2O_3;$$
$$2FeCl_2 + O + 2HCl = 2FeCl_3 + H_2O;$$
$$2FeSO_4 + 2Cl + H_2SO_4 = Fe_2(SO_4)_3 + 2HCl.$$

It is to be noticed that the normal quanities of $FeCl_2$ and $FeSO_4$ would be one-half of their formula weights if they were considered merely as a chloride and a sulphate reacting by exchanges of ions and not acting as reducing agents. It is clear, therefore, that the normal quantities of certain compounds may differ for different reactions. The equation

$$2KMnO_4 + 10FeSO_4 + 8H_2SO_4 = K_2SO_4 + 2MnSO_4 + 5Fe_2(SO_4)_3 + 8H_2O$$

indicates that one-fifth of the formula weight of $KMnO_4$ is its normal quantity in this reaction, because $2KMnO_4 \equiv 10FeSO_4$, and because $2KMnO_4 \equiv 5O$ (active). From the equation

$$H_2C_2O_4 + O(\text{active}) = H_2O + 2CO_2$$

it is to be seen that one-half the formula weight of $H_2C_2O_4$ or of $H_2C_2O_4.2H_2O$ is the normal quantity. It is evident, upon considering the formula

$$K_2Cr_2O_7 + 6FeCl_2 + 14HCl = 2KCl + 2CrCl_3 + 6FeCl_3 + 7H_2O,$$

that one-sixth of the formula weight of $K_2Cr_2O_7$ is its normal quantity; for the active oxygen in the compound shows this, as does also the amount of ferrous chloride with which it reacts.

In cases where normal solutions are too concentrated for the purposes of volumetric analysis, fractional normal solutions are frequently employed, such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{100}$ normal, etc. Normal solutions are frequently designated by the capital letter N, while the fractional solutions are represented by $\frac{1}{10}N$ or $\frac{N}{10}$, etc. The terms decinormal, centinormal, and millinormal are sometimes used to indicate $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ normal solutions. Solutions stronger than normal ones are frequently designated as $1\frac{1}{2}N$, $2N$, $3N$, etc. Solutions which bear no simple relation to normal ones are sometimes represented in normal terms by means of decimal fractions, as $.1283N$, $1.016N$, etc.

The substances equivalent to these derivatives of normal solutions may be easily found. For example, 1 ccm $\frac{1}{10}N$ sol. $\equiv$ .003545 g of Cl, .003646 g of HCl, .004904 g of H_2SO_4, etc.;

1 ccm $\frac{1}{5}N$ sol. $\equiv$ .008012 g NaOH, .01061 g of Na_2CO_3, .009808 g of H_2SO_4, etc.;

1 ccm .10091 N sol. $\equiv$ .003577 g of Cl, .010891 g of Ag, etc.

The particular advantage in the use of solutions based on the normal system is the ease with which the weight of any substance equivalent to such a solution is found.

PROBLEMS.

269. What weight of HNO_3 is contained in 1 ccm of normal HNO_3?

270. What weight of nitrogen is contained in 1 ccm of decinormal $AgNO_3$?

271. How much $\frac{1}{4}N$ acid will neutralize 20 ccm of $\frac{1}{10}N$ alkali solution?

272. What weight of MgO will 1 ccm of normal acid dissolve?

273. What weight of $BaSO_4$ will be precipitated by adding an excess of sulphuric acid to 23.5 ccm of $\frac{1}{10}N$ solution of $BaCl_2$?

274. What weight of aluminium is contained in 13.45 ccm of $\frac{1}{20}N$ $Al_2(SO_4)_3$ solution?

275. If 1 ccm of $KMnO_4 \equiv .02000$ g of Fe, when a ferrous salt is changed to a ferric one, what relation has this solution to a normal one?

276. What weight of oxygen is effective in 23.4 ccm of a $\frac{1}{4}N$ oxidizing solution?

277. What volume of $\frac{1}{10}N$ solution will react with 20.0 ccm of a .732 N solution?

278. What volume of a .1085 N solution of H_2SO_4 will neutralize 10 ccm of a $\frac{1}{10}N$ NaOH solution?

279. According to the equation

$$SnCl_2 + 2FeCl_3 = SnCl_4 + 2FeCl_2,$$

what weight of tin will be present in 1 ccm of a $\frac{1}{5}N$ solution of stannous chloride?

280. According to the equation

$$2KMnO_4 + 3MnSO_4 + 2H_2O = K_2SO_4 + 5MnO_2 + 2H_2SO_4,$$

what is the normal quantity of $KMnO_4$?

281. According to the equation given in the preceding problem, to what weight of manganese in the $MnSO_4$ will 1 ccm of the normal $KMnO_4$ solution be equivalent?

282. According to the equation

$$MnO_2 + 2FeSO_4 + 2H_2SO_4 = MnSO_4 + Fe_2(SO_4)_3 + 2H_2O,$$

to what weight of MnO_2 is 1 ccm of decinormal (reducing) $FeSO_4$ solution equivalent?

283. What weight of iodine will be set free by 1 ccm of centinormal $KMnO_4$ solution, according to the equation

$$2KMnO_4 + 10HI + 3H_2SO_4 = K_2SO_4 + 2MnSO_4 + 10I + 8H_2O?$$

284. If 50.0 ccm of a solution of HCl give by gravimetric analysis 1.2103 g of AgCl, what is the relation to normal of the HCl solution?

285. Some Na_2CO_3 is determined by acidifying it with 50.0 ccm of a .1018 *N* solution of acid, and then neutralizing the resulting solution with 10.3 ccm of an alkali, 40.0 ccm of which have been found to neutralize 37.6 ccm of the acid. What is the weight of Na_2CO_3?

286. If the nitrogen in .2341 g of a substance is converted into NH_3, and this is neutralized by 20.0 ccm of decinormal acid, what is the percentage of nitrogen in the substance?

287. If the calcium in a certain amount of $CaCO_3$ is converted into CaC_2O_4, and then 20.0 ccm of $\frac{1}{5}N$ $KMnO_4$ solution are required for decomposing this, according to the reaction

$$2KMnO_4 + 5CaC_2O_4 + 8H_2SO_4 = K_2SO_4 + 2MnSO_4 + 5CaSO_4 + 10CO_2 + 8H_2O,$$

what weight of CaO was present in the $CaCO_3$?

288. What volume of hydrogen gas will be given off by the action of 1 ccm of normal acid upon metallic zinc?

289. What volume of CO_2 will be absorbed as $BaCO_3$ by 1 ccm of normal $Ba(OH)_2$?

290. What volume of chlorine gas will be absorbed by 1 ccm of a normal reducing solution?

291. To what volume of oxygen gas does the active oxygen in 1 ccm of a normal oxidizing solution correspond?

44. The Adjustment of Volumetric Solutions. When solutions are to be prepared according to either the simple-factor or the normal system, in cases where they cannot be prepared directly by weighing the pure substances and dissolving them in definite volumes of liquid, they are usually obtained by diluting stronger solutions, which must be standardized before the adjustment is made.

If this first dilution involves uncertainty, as it frequently does when the amount of water added is considerable, and, at the same time, the volume of the

solution diluted cannot be very accurately measured, it is best to set aside a small amount of the solution, then to dilute the remainder according to calculation, and, finally, after careful standardization, to make a last adjustment by adding water or the stronger solution, as the case may require.

For example, let us suppose that a solution of NaOH is made up a little stronger than decinormal, and that comparison with .2705*N* HCl shows that 1 ccm of the latter corresponds to 2.640 ccm of the NaOH solution. It is evident that an exactly decinormal solution would have given 2.705 ccm as the result; hence the NaOH solution is $\frac{2.705}{2.640} = 1.0247$ times decinormal in its strength. A portion of the solution having been reserved, water should be added to the remainder at the rate of 2.47 ccm for each 100 ccm. Now let us assume that this first adjustment was not quite exact, but that a comparison of the diluted liquid with the .2705*N* HCl shows that 2.708 ccm of the NaOH solution neutralize 1 ccm of the acid. The solution is, therefore, too weak, and is $\frac{2.705}{2.708}$, or .9989 times decinormal. An addition of the stronger solution is therefore necessary, and it is to be observed that this will strengthen the liquid and make it decinormal, not according to the total strength, but according to its excess over the decinormal value. The deficiency of the weaker solution is .0011, and the excess of the stronger solution is .0247, in terms

of the decinormal value, so that $\frac{.0011}{.0247} \times 100 = 4.5$ ccm of the stronger solution must be added to each 100 ccm of the weaker one to make a solution of the required strength.

If the solution just considered had been too strong to the same extent as it was too weak, that is, if its strength had been 1.0011 times decinormal, an addition of .11 ccm of water to each 100 ccm would have been required.

PROBLEMS.

292. If 50.0 ccm of a solution of HCl give by gravimetric analysis .7180 g of AgCl, what volume of water must be added to 1900 ccm of the solution to make it exactly decinormal?

293. If it is found that 1 ccm of H_2SO_4 solution corresponds to .01050 g of Na_2CO_3, what volume of .2368N H_2SO_4, or what volume of water, as the case may be, must be added to 1000 of the first solution to make it $\frac{1}{5}N$?

294. If it is found that 1 ccm of a $KMnO_4$ solution is equivalent to .01015 g of Fe, how much water, or pure, solid $KMnO_4$, as the case may be, must be added to 2000 ccm of the solution in order that 1 ccm of it may correspond to .01000 g of Fe?

295. If 1 ccm of a solution of iodine is equivalent to .9287 ccm of a decinormal solution of $Na_2S_2O_3$, what volume of .1250N iodine solution, or water, as the case

may be, must be added to 1000 ccm of the first solution in order to make it exactly decinormal?

296. Two solutions, A and B, of H_2SO_4 are .09780N and .2016N. In what proportions must they be mixed to make 1000 ccm of a $\frac{1}{10}$ N solution?

45. The Simplification of Volumetric Calculations. When exactly one gram of substance, or a simple multiple or divisor of this quantity, is used for a volumetric analysis, and at the same time the factor representing the strength of the solution in terms of the substance to be determined is a simple number, the calculation of the result is very easy. For instance, if 1.000 g of a substance is taken, and 1 ccm of the solution corresponds to .01000 g of the substance to be determined, it is evident that the reading of the burette gives the percentage directly.

The weight of the substance used for analysis in the case just considered is 100 times the factor of the solution, and it is to be observed that if this relation holds in any other case, the reading of the burette must give the precentage directly. For example, if 1 ccm of a solution is equivalent to .03167 g of the substance to be determined, it is only necessary to weigh out 3.167 g of the substance to be analyzed in order that the burette reading may represent the percentage.

If the substance taken for analysis is 50 times the factor, it is then only necessary to multiply the burette reading by two to obtain the percentage, while if the weight of substance taken is 200 times the factor, the

burette reading must be divided by two to get the percentage, and so on.

This device for saving work in calculations, by weighing out such quantities of the substances to be analyzed as will effect the elimination of a multiplication by a factor and a division by the weight of substance, is particularly applicable where normal and fractional normal solutions are used, because, in these cases, the factors are to be found in a very simple manner from the formula weights of the elements or compounds to be determined.

For example, if a decinormal solution is used for the analysis of sodium carbonate containing impurities which do not neutralize an acid, the amount of substance to be taken so that the burette reading will give the percentage of Na_2CO_3 is .5305 g; if it is required to read directly the percentage of sodium in the same substance, .2305 g are taken; for Na_2O, .3105 g; for CO_2 (in a normal carbonate), .2200 g; for $Na_2CO_3.10H_2O$, 1.4313 g; for the $HNaCO_3$ that the sodium carbonate will produce, .8406 g; for the $CaCl_2.6H_2O$ which will react with it, 1.0955 g, etc.

PROBLEMS.

297. A quarter-normal solution of $K_2Cr_2O_7$ is to be used for determining iron by the oxidation of ferrous chloride to ferric chloride. What weight of substance must be taken for analysis in order that the volume

of solution multiplied by two shall give the percentage of iron?

298. With the same volumetric solution and the same process as in the preceding problem, it is desired that the burette reading shall give directly the percentage of Fe_2O_3. What weight of substance must be taken?

299. With the same volumetric solution and the same process as in the preceding two problems, it is desired to analyze a sample of ferrous carbonate, $FeCO_3$, and to have the burette reading express the percentage of FeO in the substance. What weight of the substance must be taken?

300. With a decinormal solution of ammonium thiocyanate, NH_4SCN, what weight of substance must be taken, (*a*) so that the volume of solution in cubic centimeters shall represent the percentage of silver; (*b*) so that the volume of solution used shall represent the percentage of silver sulphate? The equation of the reaction is

$$NH_4SCN + AgNO_3 = AgSCN + NH_4NO_3.$$

301. A decinormal acid solution and an alkali solution of the same strength are used in determining the ammonia produced by distilling ammonium compounds: What weight of substance must be taken, (*a*) so that the volume of acid neutralized by the ammonia may represent directly the percentage of nitrogen in the substance; (*b*) so that the volume of acid neutralized may give directly the percentage of $(NH_4)_2SO_4$ con-

tained in the substance; and (*c*) so that the volume of acid neutralized by the ammonia may represent directly the percentage of CaO, which, when converted into $Ca(OH)_2$ would be necessary to liberate the ammonia from the ammonium salt?

46. Errors in Volumetric Processes. The volume of liquid measured by a burette can be read with varying accuracy, according to the internal diameter of the instrument; that is, the *error in reading* with a slender burette is less than with a wider one. Extreme accuracy in reading, however, is usually of little importance, because the solutions are delivered drop by drop, so that the *error in delivery* may amount to nearly a drop when the most exact methods are used. Since a drop usually amounts to about .05 ccm, it is generally an unnecessary refinement to read a burette with a much greater degree of accuracy than this.

Another error connected with volumetric processes is that due to the excess of the reagent necessary to produce the end-reaction. This *error, due to the end-reaction,* varies in different cases, and it is very slight where the most delicate reactions are used. When the end-reaction is not very sharp, the amount of solution used in excess of the proper quantity is greater with more dilute volumetric solutions than with stronger ones, and it is generally increased with the dilution of the liquid in which the reaction takes place. This error may be greatly diminished by standardizing a solution by means of the substance to be determined

under conditions as nearly as possible like those used in the actual analysis, because if the error has the same proportion in the standardization and in the determination its effect is eliminated. Another way of obviating the error under consideration is to make a direct correction for the excess of solution used, after this has been determined experimentally by finding what volume of the solution is required to produce the end-reaction in the absence of the substance to be determined, or, in some cases, in the presence of a small quantity of the substance for which an allowance is made.

For careful work it is advisable to deduct from all readings of volumes the amounts used to bring about the end-reactions, if they are appreciable quantities; for when this precaution is neglected an error may arise not only in standardizing a solution, but a similar error may also be introduced in the same direction when a second solution is compared with the first one, although, of course, the two errors may counterbalance each other. For instance, if a solution A is so standardized that an excess of it amounting to one part in 500 is used for the purpose, its apparent strength will be too low in this proportion. If a solution B is then used in excess in the same proportion when it is compared with A, an additional error will be introduced, so that the apparent strength of B will be about one part in 250 less than its real strength. However, if A is used in excess in the last case, the errors will

act in opposite directions, and if equal will counterbalance each other.

Any errors that may occur in finding the volume of a solution exert a proportional effect which diminishes with the total volume used. Thus, an error of one drop, or about .05 ccm, amounts to one part in 80 when the total volume is 4 ccm, but it is only one part in 800 when the volume is 40 ccm. It is evident, therefore, that the quantities of substances used for volumetric analyses should be so regulated in connection with the strengths of the solutions employed that the volumes of these solutions should not be too small, particularly when large percentages are being determined. For example, it would be inaccurate and absurd to determine a substance amounting to 30 or 40 per cent by the use of 2 or 3 ccm of a volumetric solution, and it would be advisable to employ nearly a buretteful for the purpose.

Errors due to faulty apparatus may be briefly referred to here. It is advisable to calibrate all burettes used for accurate work, either by measuring with the burette small, equal portions of water drawn from it by means of a pipette designed for the purpose, which can be attached to the instrument, or by weighing apparently equal, small volumes of water drawn from it into a flask. Uniform graduation is the only requirement needed for a burette which is used with a solution that is not dependent upon other measuring apparatus, but when the solution has been made up in a liter-flask,

it is important that the unit of graduation of the latter should correspond to that of the burette, and a direct comparison of the two instruments should be made.

Errors depending upon faulty manipulation, such as failure to allow the walls of a burette to drain properly before reading, need not be discussed in connection with calculations.

PROBLEMS.

302. A normal acid solution is used for the determination of Na_2CO_3 in 1.00 g of substance. If the error in delivering the solution may be one drop, amounting to .05 ccm, what may be this error, expressed in terms of percentage of Na_2CO_3 in the substance?

303. What relation to normal must a solution have, in order that an error of only .1% may be produced by the difference of one drop (.05 ccm) when iron is determined in .500 g of ore?

304. What weight of substance must be used in determining available oxygen with a decinormal solution, in order that an error of .15 ccm may produce an error of only .05%?

305. If the error in the volume of a solution may be .03 ccm, what is the smallest volume of this solution that should be used in determining 1.00% of a constituent in a substance, in order that this error may not make a difference of more than .01%?

306. If an error of .2 ccm in excess has been made in standardizing a solution by use of 1.000 g of a sub-

stance and 40.2 ccm of the solution, what would be the error in percentage in determining .2500 g of the same substance where the same excess (.2 ccm) was employed, and the apparent strength of the liquid was used in the calculation?

307. If a solution *A* has been standardized in such a manner that it has been found to be stronger than it actually is, by one part in 500, and then, in comparing *B* with *A*, an excess of *A*, amounting to one part in 600, is used, what is the apparent strength of *B*, compared with its actual strength?

APPENDIX.

TABLE I.

ANSWERS TO PROBLEMS.

1. .005 and .00005
2. One part in about 33 000 and 250
3. One part in about 1200
4. About .001
5. 249.74
6. 30.06
7. 5.151
8. .02561
9. .134
10. .953
11. 42.8
12. .0025%
13. .0038 g
14. .010%
15. .13 g
16. .8 g
17. .5 g
18. Within .0002 g
19. 4 g
20. .0025%
21. .0015%
22. .005% and .001%
23. 1/60 or .017 c
24. Within .0006 g
25. .006% and .003%
26. .00035
27. .35098
28. .1656
29. 5.03
30. 1812.4
31. .9478
32. 597000
33. 4080
34. .09931
35. 38054
36. .18708
37. .00002
.00017
38. .05535
39. 107.93
40. .00994
41. 2.787
42. 2.5063
43. 1.694
44. 8.585
45. 1.308
46. 57.44% *A*
42.56% *B*
47. $A_2B_2C_7$
48. 35.45
49. 39.8
50. 12.000
51. 250.6
52. 137.4
32.05
53. 63.6
54. 81.54
55. 75.00
56. 39.14
57. *m'* 85.43
x' 126.87
58. 206.9
59. 25.47
60. 19.82
61. 41.85
62. 72.8
63. 47.1
64. K_2O, 22.03
I_2O_5, 77.97
65. 28.08 g
66. 10.99 kg CO_2
67. 15.28 oz. $AgNO_3$
68. 35.5 g P_2O_5
69. 2447 lbs.
70. 25.47 g
71. .6829
72. 1.359
73. .7898
74. .4827
75. 2.698
76. 1 111
77. 137.3
78. 30.17
79. 1.816 g
80. 11.74 g
81. 8.255 g
82. 7.771 oz.
83. 7.386 kg
84. 13.54 g
85. 115.3 g
86. 118.3 g
87. 15.28 g
88. 3.431 g
89. 8.060
90. 36.72%
91. 7.31%
92. 1.692%
93. 27.14%
94. .1373 g
95. 2.784 g
96. 75.28 g
97. 1.149 g
98. .7200 g
99. 2.520 g

TABLE I.—*Continued.*

100. 50
101. 9.35% K, 11.58% Na
102. 5.03% K_2O, 9.92% Na_2O
103. 5.00% Fe, 6.82% Al
104. 16.53% Fe, 8.53% PO_4
105. 6.83% ZnS, 21.47% FeS_2
106. 20.53% SrO, 5.60% CaO
107. 1.3873
108. 2.784
109. 17.96% Br, 9.34% Cl
110. .0815 g KCl
111. .0700 g $CaSO_4$
112. .5858 g AgBr, .4142 g AgI
113. 5000 g Na_2SO_4, 5000 g Li_2SO_4
114. .0621 g Na_2SO_4, .0379 g K_2SO_4
115. .0426 g Br
116. .5405 g I
117. .1001 g $CaCO_3$
118. $MgCl_2.6H_2O$
119. $K_2Fe(SO_4)_2.6H_2O$
120. C_6H_7N
121. $C_9H_{10}O_2$
122. KPb_2Br_5
123. $KPbI_3.2H_2O$
124. $H_2Rb_3I_2ClO_6$
125. $Pb_2I_5H_2O_2$
126. $Cu(NO_3)_2.3Cu(OH)_2$
127. HMn_2PO_5
128. $CaMnSiO_4$
129. $KPb_2(Br,I)_5$
130. $Cs(Cl,Br)_2I$
131. $K\,Pb_2(Br,I)_8.4H_2O$
132. $M''AlPO_5.2H_2O$ ($M''=Fe'',Mn'',Ca'',Na_2''$)
133. (Li,Na)(Mn,Fe)PO_4
134. (Hg,Cd)(Se,S)
135. BeCa(F,OH)PO_4
136. $AlCu_6ClSO_{10}.9H_2O$
137. $H_2R_4'Al_4(SiO_3)_9$ ($R'=Cs,K,Na,Li$)
138. $H(Al,Fe''')_5(Fe,Mn,Mg)Si_2O_{13}$
139. $Ag_8(Sn,Ge)S_6$
140. $R''Al_2Si_3O_{10}.3H_2O$ ($R''=Ba,Sr,Ca,Mg,K_2Na_2$)
141. $H_2CaZnSiO_5$
142. 50.29%
143. 47.95%
144. 55.90%
145. 19.82
146. $NH_4Fe(SO_4)_2.12H_2O$
147. $Cs_2Cu_2Cl_7.2H_2O$
148. 70.25% NaCl, 10.00% KCl, 4.75% K_2SO_4, 15.00% $MgSO_4.7H_2O$
149. C_2N_2
150. C_5H_6O
151. C_8H_{18}
152. $C_8H_8NCl_2$
153. $\frac{2}{3}$ cu. ft.
154. 152 mm
155. $13\frac{1}{3}$ cu. ft.
156. $814\frac{2}{7}$ mm
157. 92.1 ccm
158. 615 mm
159. 117.8 ccm
160. 136.6 ccm
161. −146.5°
162. 111.9 ccm
163. 457°
164. 150 ccm
165. −173°
166. −219.7°
167. 1.99 atm.
168. 58.5°
169. 87.5 ccm
170. 50.3°
171. 29.9 in.
172. 26.0 ccm
173. .000958 g
174. .01192 g
175. .00152 g
176. .00280 g
177. 1½:1
178. 1:4.325
179. 28
180. 1.52
181. 100.0
182. 4.66 g
183. 36.47
184. About 70
185. About 2.0 g
186. 72
187. 18.12
188. .00289 g
189. 98.64
190. 54.70 ccm
191. 2128 mm

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TABLE I.—*Continued.*

192. 178.6
193. 8495 ccm
194. .8167
195. 22.4 l
196. 200 ccm
197. 15 ccm
198. 10 000 ccm
199. 15.03 l
200. 2000 ccm
201. 327°
202. 5.63 atm.
203. 8.37 l
204. 21.25 l
205. 36.5 atm.
206. 605° C.
207. 23.5 kg Fe
82.4 kg 50% H_2SO_4
208. 12.9 g
209. 3.43 l
210. 9.57 l
211. 92.7%
212. 92.9%
213. 2567 l
214. 72.9 l
215. 39.1 cu. ft.
216. 3.68 lbs.
217. 11.14 oz.
218. 753 lbs.
219. 2.22 lbs.
220. 4.47 cu. ft.
221. 803° F.
222. 18.5 ccm
223. 47.1 ccm
224. .00039 g
225. .00486 g
226. 3.32%
227. 1262 kg
228. 1.67%
73%
229. .001146 g
230. 1.44%
231. 752.2 mm
232. 29.87 in.
233. 763.2 mm
234. $1\frac{1}{2}$ times as great
235. .00635 g
236. .0000950 g
237. .006246 g
238. .005336 g
239. 2.905 g
240. 4.207 g
241. .01706 g
242. .08014 g
243. .01466 g
244. .006143 g
245. .007116 g
246. .0895 g
247. .6073 g
248. .2275 g
249. 5.068%
250. 45.56%
251. 7.18%
252. 12.39 ccm
253. 22.16 ccm
254. .02510 g
255. .006944 g
256. .004377 g
257. .003354 g
258. .448 g
259. .04864 g
260. .008991 g
261. 5.80%
262. 27.87%
263. .2435 g
264. .806 g
265. 60.15%
266. 51.45%
267. 17.561 g
268. .03973 g
269. .06305 g
270. .001404 g
271. 8 ccm
272. .02018
273. .2744 g
274. .00607 g
275. .3578 N
276. .0468 g
277. 146.4 ccm
278. 9.22 ccm
279. .0119 g
280. 52.72 g
281. .0275 g
282. .00435 g
283. .0012697 g
284. .1688 N
285. .2177 g
286. 11.99%
287. .1122 g
288. 11.20 ccm
289. 11.20 ccm
290. 11.20 ccm
291. 5.60 ccm
292. 2.9 ccm
293. 56.4 ccm
294. 30 ccm
295. 285.2 ccm
296. A. 978.8 ccm
B. 21.2 ccm
297. .6988 g
298. 1.998
299. 1.798
300. (*a*) 1.0793 g
(*b*) 1.5596 g
301. (*a*) .1404 g
(*b*) .6610 g
(*c*) .2805 g
302. .27%
303. .18 N
304. .24 g
305. 3.0 ccm
306. +.37%
307. $\frac{1}{5000}$ too strong

TABLE II.

MEASURES AND WEIGHTS.

LENGTH.

1000 millimeters (mm) = 100 centimeters (cm) = 1 meter (m) = 3.2808 ft.

10 millimeters (mm) = 1 centimeter = .39370 inch

1 foot = .30480 m 1 inch = 2.540 cm

AREA.

10 000 sq. centimeters (scm) = 1 sq. meter (sm) = 10.764 sq. ft.

1 sq. centimeter (scm) = .1550 sq. in.

1 sq. foot = .09290 sm 1 sq. inch = 6.4516 scm

VOLUME.

1 000 000 cu. centimeters (ccm) = 1000 liters (l) = 1 cu. meter = 35.314 cu. ft.

1 000 cu. centimeters (ccm) = 1 cu. decimeter = 1 liter (l) = 61.02 cu. in.

1 cu. centimeter = .06102 cu. in.

1 cu. foot = .02832 cu. meter 1 cu. inch = 16.387 ccm

CAPACITY.

1000 cu. centimeters (ccm) = 1 liter (l) = 1.0567 U. S. quarts = .26417 U. S. gallons

1 cu. centimeter (ccm) = .03381 U. S. fluid ounces = .06102 cu. in.

1 U. S. fluid ounce = 29.574 ccm 1 U. S. quart = .94635 l

1 U. S. gallon = 3.7854 l

WEIGHT.

1 000 000 milligrams (mg) = 1000 grams (g) = 1 kilogram (kg) = 2.2046 lbs. av.

1 000 milligrams (mg) = 1 gram (g) = 15.432 grains

1 gram (g) = .03215 oz. troy = .03527 oz. av.

1 grain = .06480 g 1 oz. troy = 31.103 g 1 oz. av. = 28.350 g

1 lb. av. = 453.59 g

TABLE III.

PRESSURE OF WATER-VAPOR.*

(From Broch's Calculations of Regnault's Results.)

A.

Temp. °C.	Press. mm.	Temp. °C.	Press. mm.
0	4.6	16	13.5
0.5	4.7	16.5	13.9
1	4.9	17	14.4
1.5	5.1	17.5	14.9
2	5.3	18	15.3
2.5	5.5	18.5	15.8
3	5.7	19	16.3
3.5	5.9	19.5	16.8
4	6.1	20	17.4
4.5	6.3	20.5	17.9
5	6.5	21	18.5
5.5	6.7	21.5	19.0
6	7.0	22	19.6
6.5	7.2	22.5	20.2
7	7.5	23	20.9
7.5	7.7	23.5	21.5
8	8.0	24	22.2
8.5	8.3	24.5	22.8
9	8.5	25	23.5
9.5	8.8	25.5	24.2
10	9.1	26	25.0
10.5	9.4	26.5	25.7
11	9.8	27	26.5
11.5	10.1	27.5	27.3
12	10.4	28	28.1
12.5	10.8	28.5	28.9
13	11.1	29	29.7
13.5	11.5	29.5	30.6
14	11.9	30	31.5
14.5	12.3	30.5	32.4
15	12.7	31	33.4
15.5	13.1	31.5	34.3

B.

Temp. °C.	Press. mm.	Temp. °C.	Press. mm.	Temp. °C.	Press. mm.	Temp. °C.	Press. mm.
−19	1.03	12	10.43	43	64.31	74	276.87
−18	1.12	13	11.14	44	67.76	75	288.76
−17	1.22	14	11.88	45	71.36	76	301.09
−16	1.32	15	12.67	46	75.13	77	313.85
−15	1.44	16	13.51	47	79.07	78	327.05
−14	1.56	17	14.40	48	83.19	79	340.73
−13	1.69	18	15.33	49	87.49	80	354.87
−12	1.84	19	16.32	50	91.98	81	369.51
−11	1.99	20	17.36	51	96.66	82	384.64
−10	2.15	21	18.47	52	101.55	83	400.29
− 9	2.33	22	19.63	53	106.65	84	416.47
− 8	2.51	23	20.86	54	111.97	85	433.19
− 7	2.72	24	22.15	55	117.52	86	450.47
− 6	2.93	25	23.52	56	123.29	87	468.32
− 5	3.16	26	24.96	57	129.31	88	486.76
− 4	3.41	27	26.47	58	135.58	89	505.81
− 3	3.67	28	28.07	59	142.10	90	525.47
− 2	3.95	29	29.74	60	148.88	91	545.77
− 1	4.25	30	31.51	61	155.95	92	566.71
0	4.57	31	33.37	62	163.29	93	588.33
+ 1	4.91	32	35.32	63	170.92	94	610.64
2	5.27	33	37.37	64	178.86	95	633.66
3	5.66	34	39.52	65	187.10	96	657.40
4	6.07	35	41.78	66	195.67	97	681.88
5	6.51	36	44.16	67	204.56	98	707.13
6	6.97	37	46.65	68	213.79	99	733.16
7	7.47	38	49.26	69	223.37	100	760.00
8	7.99	39	52.00	70	233.31	101	787.67
9	8.55	40	54.87	71	243.62	102	815.99
10	9.14	41	57.87	72	254.30		
11	9.77	42	61.02	73	265.38		

* The table marked A is sufficiently accurate for the calculations connected with the problems given in this book, and for all ordinary work with gases, as far as its range extends.

TABLE IV.

ATOMIC WEIGHTS.

(International Table of 1905.)

O = 16.

Aluminium	Al	27.1	Neon	Ne	20
Antimony	Sb	120.2	Nickel	Ni	58.7
Argon	A	39.9	Niobium (Columbium)	Nb	94
Arsenic	As	75.0	Nitrogen	N	14.04
Barium	Ba	137.4	Osmium	Os	191
Beryllium (Glucinum)	Be	9.1	Oxygen	O	16.00
Bismuth	Bi	208.5	Palladium	Pd	106.5
Boron	B	11.0	Phosphorus	P	31.0
Bromine	Br	79.96	Platinum	Pt	194.8
Cadmium	Cd	112.4	Potassium	K	39.15
Cæsium	Cs	132.9	Praseodymium	Pr	140.5
Calcium	Ca	40.1	Radium	Ra	225
Carbon	C	12.00	Rhodium	Rh	103.0
Cerium	Ce	140.25	Rubidium	Rb	85.5
Chlorine	Cl	35.45	Ruthenium	Ru	101.7
Chromium	Cr	52.1	Samarium	Sm	150.3
Cobalt	Co	59.0	Scandium	Sc	44.1
Copper	Cu	63.6	Selenium	Se	79.2
Erbium	E	166	Silicon	Si	28.4
Fluorine	F	19	Silver	Ag	107.93
Gadolinium	Gd	156	Sodium	Na	23.05
Gallium	Ga	70	Strontium	Sr	87.6
Germanium	Ge	72.5	Sulphur	S	32.06
Gold	Au	197.2	Tantalum	Ta	183
Helium	He	4	Tellurium	Te	127.6
Hydrogen	H	1.008	Terbium	Tb	160
Indium	In	115	Thallium	Tl	204.1
Iodine	I	126.97	Thorium	Th	232.5
Iridium	Ir	193.0	Thulium	Tm	171
Iron	Fe	55.9	Tin	Sn	119.0
Krypton	Kr	81.8	Titanium	Ti	48.1
Lanthanum	La	138.9	Tungsten	W	184
Lead	Pb	206.9	Uranium	U	238.5
Lithium	Li	7.03	Vanadium	V	51.2
Magnesium	Mg	24.36	Xenon	X	128
Manganese	Mn	55.0	Ytterbium	Yb	173.0
Mercury	Hg	200.0	Yttrium	Yt	89.0
Molybdenum	Mo	96.0	Zinc	Zn	65.4
Neodymium	Nd	143.6	Zirconium	Zr	90.6

TABLE V.

ATOMIC WEIGHTS OF COMMONER ELEMENTS AND SOME OF THEIR MULTIPLES, WITH LOGARITHMS.

Element.	At. Wt.	Log.	Element.	At. Wt.	Log.
Ag	107.93	.033314	I	126.97	.10370
2Ag	215.86	.33417	2I	253.94	.40473
Al	27.1	.43297	K	39.15	.59273
2Al	54.2	.73400	2K	78.30	.89376
As	75.0	.87506	Li	7.03	.84696
2As	150.0	.17609	Mg	24.36	.38668
Au	197.2	.29491	Mn	55.0	.74036
B	11.0	.04139	N	14.04	.14737
Ba	137.4	.13799	2N	28.08	.44840
Bi	208.5	.31911	Na	23.05	.36267
Br	79.96	.90287	2Na	46.10	.66370
2Br	159.92	.20390	Ni	58.7	.76864
C	12.00	.07918	O	16.00	.20412
Ca	40.1	.60314	2O	32.00	.50515
Cd	112.4	.05077	3O	48.00	.68124
Cl	35.45	.54962	P	31.0	.49136
2Cl	70.90	.85065	2P	62.0	.79239
3Cl	106.35	.02674	Pb	206.9	.31576
Co	59.0	.77085	Pt	194.8	.28959
Cr	52.1	.71684	S	32.06	.50596
2Cr	104.2	.01787	2S	64.12	.80699
Cu	63.6	.80346	Sb	120.2	.07990
Fe	55.9	.74741	Si	28.4	.45332
2Fe	111.8	.04844	Sn	119.0	.07555
H	1.008	.00346	Sr	87.6	.94250
2H	2.016	.30449	Zn	65.4	.81558
3H	3.024	.48058			
4H	4.032	.60552			

TABLE VI.

SOME MULTIPLES OF ATOMIC WEIGHTS AND GROUP WEIGHTS.

Br,	79.96	4Cl,	141.80	I,	126.97	6O,	96.00
2Br,	159.92	5Cl,	177.25	2I,	253.94	7O,	112.00
3Br,	239.88	6Cl,	212.70	3I,	380.91	8O,	128.00
4Br,	319.84			4I,	507.9	9O,	144.00
5Br,	399.80	H,	1.008	5I,	634.9		
6Br,	479.76	2H,	2.016	6I,	761.8	OH,	17.008
		3H,	3.024			2OH,	34.016
C,	12.00	4H,	4.032	N,	14.04	3OH,	51.024
2C,	24.00	5H,	5.040	2N,	28.08		
3C,	36.00	6H,	6.048	3N,	42.12	PO_4,	95.0
4C,	48.00	7H,	7.056	4N,	56.16	P_2O_7,	174.0
5C,	60.00	8H,	8.064	5N,	70.20		
6C,	72.00	9H,	9.072	6N,	84.24	S,	32.06
7C,	84.00					2S,	64.12
8C,	96.00	H_2O,	18.016	NH_4,	18.072	3S,	96.18
9C,	108.00	$2H_2O$,	36.032	$2NH_4$,	36.144	4S,	128.24
		$3H_2O$,	54.048			5S,	160.30
CN,	26.04	$4H_2O$,	72.064	NO_3,	62.04		
2CN,	52.08	$5H_2O$,	90.080	$2NO_3$,	124.08	SO_4,	96.06
6CN,	156.24	$6H_2O$,	108.10	$3NO_3$,	186.12	$2SO_4$,	192.12
		$7H_2O$,	126.11			$3SO_4$,	288.18
CO_3,	60.00	$8H_2O$,	144.13	O,	16.00		
		$9H_2O$,	162.14	2O,	32.00		
Cl,	35.45	$10H_2O$,	180.16	3O,	48.00		
2Cl,	70.90	$11H_2O$,	198.18	4O,	64.00		
3Cl,	106.35	$12H_2O$,	216.19	5O,	80.00		

TABLE VII.

LOGARITHMS OF NUMBERS.

(Electrotypes from Carhart's "Field Book for Civil Engineers.")

N	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
1	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
2	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
3	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
4	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
5	2119	2160	2202	2243	2284	2325	2366	2407	2449	2490
6	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
7	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
8	3342	3383	3426	3463	3503	3543	3583	3623	3663	3703
9	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
1	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
2	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
3	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
4	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
5	6070	6108	6145	6183	6221	6258	6296	6333	6371	6408
6	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
7	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
8	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
9	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
1	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
2	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
3	8991	9036	9061	9096	9132	9167	9202	9237	9272	9307
4	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
5	9691	9726	9760	9795	9830	9864	9899	9934	9968	10003
6	10037	10072	10106	10140	10175	10209	10243	10278	10312	0346
7	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
8	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
9	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
1	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
2	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
3	2385	2418	2450	2483	2516	2548	2581	2613	2646	2678
4	2710	2743	2775	2808	2840	2872	2905	2937	2969	3001
5	3033	3066	3098	3130	3162	3194	3226	3258	3290	3322
6	3354	3386	3418	3450	3481	3513	3545	3577	3609	3640
7	3672	3704	3735	3767	3799	3830	3862	3893	3925	3956
8	3988	4019	4051	4082	4114	4145	4176	4208	4239	4270
9	4301	4333	4364	4395	4426	4457	4489	4520	4551	4582
140	14613	14644	14675	14706	14737	14768	14799	14829	14860	14891
1	4922	4953	4983	5014	5045	5076	5106	5137	5168	5198
2	5229	5259	5290	5320	5351	5381	5412	5442	5473	5503
3	5534	5564	5594	5625	5655	5685	5715	5746	5776	5806
4	5836	5866	5897	5927	5957	5987	6017	6047	6077	6107
5	6137	6167	6197	6227	6256	6286	6316	6346	6376	6406
6	6435	6465	6495	6524	6554	6584	6613	6643	6673	6702
7	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
8	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
9	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
150	17609	17638	17667	17696	17725	17754	17782	17811	17840	17869
1	7898	7926	7955	7984	8013	8041	8070	8099	8127	8156
2	8184	8213	8241	8270	8298	8327	8355	8384	8412	8441
3	8469	8498	8526	8554	8583	8611	8639	8667	8696	8724
4	8752	8780	8808	8837	8865	8893	8921	8949	8977	9005
5	9033	9061	9089	9117	9145	9173	9201	9229	9257	9285
6	9312	9340	9368	9396	9424	9451	9479	9507	9535	9562
7	9590	9618	9645	9673	9700	9728	9756	9783	9811	9838
8	9866	9893	9921	9948	9976	20003	20030	20058	20085	20112
9	20140	20167	20194	20222	20249	0276	0303	0330	0358	0385
160	20412	20439	20466	20493	20520	20548	20575	20602	20629	20656
1	0683	0710	0737	0763	0790	0817	0844	0871	0898	0925
2	0952	0978	1005	1032	1059	1085	1112	1139	1165	1192
3	1219	1245	1272	1299	1325	1352	1378	1405	1431	1458
4	1484	1511	1537	1564	1590	1617	1643	1669	1696	1722
5	1748	1775	1801	1827	1854	1880	1906	1932	1958	1985
6	2011	2037	2063	2089	2115	2141	2167	2194	2220	2246
7	2272	2298	2324	2350	2376	2401	2427	2453	2479	2505
8	2531	2557	2583	2608	2634	2660	2686	2712	2737	2763
9	2789	2814	2840	2866	2891	2917	2943	2968	2994	3019
170	23045	23070	23096	23121	23147	23172	23198	23223	23249	23274
1	3300	3325	3350	3376	3401	3426	3452	3477	3502	3528
2	3553	3578	3603	3629	3654	3679	3704	3729	3754	3779
3	3805	3830	3855	3880	3905	3930	3955	3980	4005	4030
4	4055	4080	4105	4130	4155	4180	4204	4229	4254	4279
5	4304	4329	4353	4378	4403	4428	4452	4477	4502	4527
6	4551	4576	4601	4625	4650	4674	4699	4724	4748	4773
7	4797	4822	4846	4871	4895	4920	4944	4969	4993	5018
8	5042	5066	5091	5115	5139	5164	5188	5212	5237	5261
9	5285	5310	5334	5358	5382	5406	5431	5455	5479	5503
180	25527	25551	25575	25600	25624	25648	25672	25696	25720	25744
1	5768	5792	5816	5840	5864	5888	5912	5935	5959	5983
2	6007	6031	6055	6079	6102	6126	6150	6174	6198	6221
3	6245	6269	6293	6316	6340	6364	6387	6411	6435	6458
4	6482	6505	6529	6553	6576	6600	6623	6647	6670	6694
5	6717	6741	6764	6788	6811	6834	6858	6881	6905	6928
6	6951	6975	6998	7021	7045	7068	7091	7114	7138	7161
7	7184	7207	7231	7254	7277	7300	7323	7346	7370	7393
8	7416	7439	7462	7485	7508	7531	7554	7577	7600	7623
9	7646	7669	7692	7715	7738	7761	7784	7807	7830	7852
190	27875	27898	27921	27944	27967	27989	28012	28035	28058	28081
1	8103	8126	8149	8171	8194	8217	8240	8262	8285	8307
2	8330	8353	8375	8398	8421	8443	8466	8488	8511	8533
3	8556	8578	8601	8623	8646	8668	8691	8713	8735	8758
4	8780	8803	8825	8847	8870	8892	8914	8937	8959	8981
5	9003	9026	9048	9070	9092	9115	9137	9159	9181	9203
6	9226	9248	9270	9292	9314	9336	9358	9380	9403	9425
7	9447	9469	9491	9513	9535	9557	9579	9601	9623	9645
8	9667	9688	9710	9732	9754	9776	9798	9820	9842	9863
9	9885	9907	9929	9951	9973	9994	30016	30038	30060	30081
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
200	30103	30125	30146	30168	30190	30211	30233	30255	30276	30298
1	0320	0341	0363	0384	0406	0428	0449	0471	0492	0514
2	0535	0557	0578	0600	0621	0643	0664	0685	0707	0728
3	0750	0771	0792	0814	0835	0856	0878	0899	0920	0942
4	0963	0984	1006	1027	1048	1069	1091	1112	1133	1154
5	1175	1197	1218	1239	1260	1281	1302	1323	1345	1366
6	1387	1408	1429	1450	1471	1492	1513	1534	1555	1576
7	1597	1618	1639	1660	1681	1702	1723	1744	1765	1785
8	1806	1827	1848	1869	1890	1911	1931	1952	1973	1994
9	2015	2035	2056	2077	2098	2118	2139	2160	2181	2201
210	32222	32243	32263	32284	32305	32325	32346	32366	32387	32408
1	2428	2449	2469	2490	2510	2531	2552	2572	2593	2613
2	2634	2654	2675	2695	2715	2736	2756	2777	2797	2818
3	2838	2858	2879	2899	2919	2940	2960	2980	3001	3021
4	3041	3062	3082	3102	3122	3143	3163	3183	3203	3224
5	3244	3264	3284	3304	3325	3345	3365	3385	3405	3425
6	3445	3465	3486	3506	3526	3546	3566	3586	3606	3626
7	3646	3666	3686	3706	3726	3746	3766	3786	3806	3826
8	3846	3866	3885	3905	3925	3945	3965	3985	4005	4025
9	4044	4064	4084	4104	4124	4143	4163	4183	4203	4223
220	34242	34262	34282	34301	34321	34341	34361	34380	34400	34420
1	4439	4459	4479	4498	4518	4537	4557	4577	4596	4616
2	4635	4655	4674	4694	4713	4733	4753	4772	4792	4811
3	4830	4850	4869	4889	4908	4928	4947	4967	4986	5005
4	5025	5044	5064	5083	5102	5122	5141	5160	5180	5199
5	5218	5238	5257	5276	5295	5315	5334	5353	5372	5392
6	5411	5430	5449	5468	5488	5507	5526	5545	5564	5583
7	5603	5622	5641	5660	5679	5698	5717	5736	5755	5774
8	5793	5813	5832	5851	5870	5889	5908	5927	5946	5965
9	5984	6003	6021	6040	6059	6078	6097	6116	6135	6154
230	36173	36192	36211	36229	36248	36267	36286	36305	36324	36342
1	6361	6380	6399	6418	6436	6455	6474	6493	6511	6530
2	6549	6568	6586	6605	6624	6642	6661	6680	6698	6717
3	6736	6754	6773	6791	6810	6829	6847	6866	6884	6903
4	6922	6940	6959	6977	6996	7014	7033	7051	7070	7088
5	7107	7125	7144	7162	7181	7199	7218	7236	7254	7273
6	7291	7310	7328	7346	7365	7383	7401	7420	7438	7457
7	7475	7493	7511	7530	7548	7566	7585	7603	7621	7639
8	7658	7676	7694	7712	7731	7749	7767	7785	7803	7822
9	7840	7858	7876	7894	7912	7931	7949	7967	7985	8003
240	38021	38039	38057	38075	38093	38112	38130	38148	38166	38184
1	8202	8220	8238	8256	8274	8292	8310	8328	8346	8364
2	8382	8399	8417	8435	8453	8471	8489	8507	8525	8543
3	8561	8578	8596	8614	8632	8650	8668	8686	8703	8721
4	8739	8757	8775	8792	8810	8828	8846	8863	8881	8899
5	8917	8934	8952	8970	8987	9005	9023	9041	9058	9076
6	9094	9111	9129	9146	9164	9182	9199	9217	9235	9252
7	9270	9287	9305	9322	9340	9358	9375	9393	9410	9428
8	9445	9463	9480	9498	9515	9533	9550	9568	9585	9602
9	9620	9637	9655	9672	9690	9707	9724	9742	9759	9777
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
250	39794	39811	39829	39846	39863	39881	39898	39915	39933	39950
1	9967	9985	40002	40019	40037	40054	40071	40088	40106	40123
2	40140	40157	0175	0192	0209	0226	0243	0261	0278	0295
3	0312	0329	0346	0364	0381	0398	0415	0432	0449	0466
4	0483	0500	0518	0535	0552	0569	0586	0603	0620	0637
5	0654	0671	0688	0705	0722	0739	0756	0773	0790	0807
6	0824	0841	0858	0875	0892	0909	0926	0943	0960	0976
7	0993	1010	1027	1044	1061	1078	1095	1111	1128	1145
8	1162	1179	1196	1212	1229	1246	1263	1280	1296	1313
9	1330	1347	1363	1380	1397	1414	1430	1447	1464	1481
260	41497	41514	41531	41547	41564	41581	41597	41614	41631	41647
1	1664	1681	1697	1714	1731	1747	1764	1780	1797	1814
2	1830	1847	1863	1880	1896	1913	1929	1946	1963	1979
3	1996	2012	2029	2045	2062	2078	2095	2111	2127	2144
4	2160	2177	2193	2210	2226	2243	2259	2275	2292	2308
5	2325	2341	2357	2374	2390	2406	2423	2439	2455	2472
6	2488	2504	2521	2537	2553	2570	2586	2602	2619	2635
7	2651	2667	2684	2700	2716	2732	2749	2765	2781	2797
8	2813	2830	2846	2862	2878	2894	2911	2927	2943	2959
9	2975	2991	3008	3024	3040	3056	3072	3088	3104	3120
270	43136	43152	43169	43185	43201	43217	43233	43249	43265	43281
1	3297	3313	3329	3345	3361	3377	3393	3409	3425	3441
2	3457	3473	3489	3505	3521	3537	3553	3569	3584	3600
3	3616	3632	3648	3664	3680	3696	3712	3727	3743	3759
4	3775	3791	3807	3823	3838	3854	3870	3886	3902	3917
5	3933	3949	3965	3981	3996	4012	4028	4044	4059	4075
6	4091	4107	4122	4138	4154	4170	4185	4201	4217	4232
7	4248	4264	4279	4295	4311	4326	4342	4358	4373	4389
8	4404	4420	4436	4451	4467	4483	4498	4514	4529	4545
9	4560	4576	4592	4607	4623	4638	4654	4669	4685	4700
280	44716	44731	44747	44762	44778	44793	44809	44824	44840	44855
1	4871	4886	4902	4917	4932	4948	4963	4979	4994	5010
2	5025	5040	5056	5071	5086	5102	5117	5133	5148	5163
3	5179	5194	5209	5225	5240	5255	5271	5286	5301	5317
4	5332	5347	5362	5378	5393	5408	5423	5439	5454	5469
5	5484	5500	5515	5530	5545	5561	5576	5591	5606	5621
6	5637	5652	5667	5682	5697	5712	5728	5743	5758	5773
7	5788	5803	5818	5834	5849	5864	5879	5894	5909	5924
8	5939	5954	5969	5984	6000	6015	6030	6045	6060	6075
9	6090	6105	6120	6135	6150	6165	6180	6195	6210	6225
290	46240	46255	46270	46285	46300	46315	46330	46345	46359	46374
1	6389	6404	6419	6434	6449	6464	6479	6494	6509	6523
2	6538	6553	6568	6583	6598	6613	6627	6642	6657	6672
3	6687	6702	6716	6731	6746	6761	6776	6790	6805	6820
4	6835	6850	6864	6879	6894	6909	6923	6938	6953	6967
5	6982	6997	7012	7026	7041	7056	7070	7085	7100	7114
6	7129	7144	7159	7173	7188	7202	7217	7232	7246	7261
7	7276	7290	7305	7319	7334	7349	7363	7378	7392	7407
8	7422	7436	7451	7465	7480	7494	7509	7524	7538	7553
9	7567	7582	7596	7611	7625	7640	7654	7669	7683	7698
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
300	47712	47727	47741	47756	47770	47784	47799	47813	47828	47842
1	7857	7871	7885	7900	7914	7929	7943	7958	7972	7986
2	8001	8015	8029	8044	8058	8073	8087	8101	8116	8130
3	8144	8159	8173	8187	8202	8216	8230	8244	8259	8273
4	8287	8302	8316	8330	8344	8359	8373	8387	8401	8416
5	8430	8444	8458	8473	8487	8501	8515	8530	8544	8558
6	8572	8586	8601	8615	8629	8643	8657	8671	8686	8700
7	8714	8728	8742	8756	8770	8785	8799	8813	8827	8841
8	8855	8869	8883	8897	8911	8926	8940	8954	8968	8982
9	8996	9010	9024	9038	9052	9066	9080	9094	9108	9122
310	49136	49150	49164	49178	49192	49206	49220	49234	49248	49262
1	9276	9290	9304	9318	9332	9346	9360	9374	9388	9402
2	9415	9429	9443	9457	9471	9485	9499	9513	9527	9541
3	9554	9568	9582	9596	9610	9624	9638	9651	9665	9679
4	9693	9707	9721	9734	9748	9762	9776	9790	9803	9817
5	9831	9845	9859	9872	9886	9900	9914	9927	9941	9955
6	9969	9982	9996	50010	50024	50037	50051	50065	50079	50092
7	50106	50120	50133	0147	0161	0174	0188	0202	0215	0229
8	0243	0256	0270	0284	0297	0311	0325	0338	0352	0365
9	0379	0393	0406	0420	0433	0447	0461	0474	0488	0501
320	50515	50529	50542	50556	50569	50583	50596	50610	50623	50637
1	0651	0664	0678	0691	0705	0718	0732	0745	0759	0772
2	0786	0799	0813	0826	0840	0853	0866	0880	0893	0907
3	0920	0934	0947	0961	0974	0987	1001	1014	1028	1041
4	1055	1068	1081	1095	1108	1121	1135	1148	1162	1175
5	1188	1202	1215	1228	1242	1255	1268	1282	1295	1308
6	1322	1335	1348	1362	1375	1388	1402	1415	1428	1441
7	1455	1468	1481	1495	1508	1521	1534	1548	1561	1574
8	1587	1601	1614	1627	1640	1654	1667	1680	1693	1706
9	1720	1733	1746	1759	1772	1786	1799	1812	1825	1838
330	51851	51865	51878	51891	51904	51917	51930	51943	51957	51970
1	1983	1996	2009	2022	2035	2048	2061	2075	2088	2101
2	2114	2127	2140	2153	2166	2179	2192	2205	2218	2231
3	2244	2257	2270	2284	2297	2310	2323	2336	2349	2362
4	2375	2388	2401	2414	2427	2440	2453	2466	2479	2492
5	2504	2517	2530	2543	2556	2569	2582	2595	2608	2621
6	2634	2647	2660	2673	2686	2699	2711	2724	2737	2750
7	2763	2776	2789	2802	2815	2827	2840	2853	2866	2879
8	2892	2905	2917	2930	2943	2956	2969	2982	2994	3007
9	3020	3033	3046	3058	3071	3084	3097	3110	3122	3135
340	53148	53161	53173	53186	53199	53212	53224	53237	53250	53263
1	3275	3288	3301	3314	3326	3339	3352	3364	3377	3390
2	3403	3415	3428	3441	3453	3466	3479	3491	3504	3517
3	3529	3542	3555	3567	3580	3593	3605	3618	3631	3643
4	3656	3668	3681	3694	3706	3719	3732	3744	3757	3769
5	3782	3794	3807	3820	3832	3845	3857	3870	3882	3895
6	3908	3920	3933	3945	3958	3970	3983	3995	4008	4020
7	4033	4045	4058	4070	4083	4095	4108	4120	4133	4145
8	4158	4170	4183	4195	4208	4220	4233	4245	4258	4270
9	4283	4295	4307	4320	4332	4345	4357	4370	4382	4394
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
350	54407	54419	54432	54444	54456	54469	54481	54494	54506	54518
1	4531	4543	4555	4568	4580	4593	4605	4617	4630	4642
2	4654	4667	4679	4691	4704	4716	4728	4741	4753	4765
3	4777	4790	4802	4814	4827	4839	4851	4864	4876	4888
4	4900	4913	4925	4937	4949	4962	4974	4986	4998	5011
5	5023	5035	5047	5060	5072	5084	5096	5108	5121	5133
6	5145	5157	5169	5182	5194	5206	5218	5230	5242	5255
7	5267	5279	5291	5303	5315	5328	5340	5352	5364	5376
8	5388	5400	5413	5425	5437	5449	5461	5473	5485	5497
9	5509	5522	5534	5546	5558	5570	5582	5594	5606	5618
360	55630	55642	55654	55666	55678	55691	55703	55715	55727	55739
1	5751	5763	5775	5787	5799	5811	5823	5835	5847	5859
2	5871	5883	5895	5907	5919	5931	5943	5955	5967	5979
3	5991	6003	6015	6027	6038	6050	6062	6074	6086	6098
4	6110	6122	6134	6146	6158	6170	6182	6194	6205	6217
5	6229	6241	6253	6265	6277	6289	6301	6312	6324	6336
6	6348	6360	6372	6384	6396	6407	6419	6431	6443	6455
7	6467	6478	6490	6502	6514	6526	6538	6549	6561	6573
8	6585	6597	6608	6620	6632	6644	6656	6667	6679	6691
9	6703	6714	6726	6738	6750	6761	6773	6785	6797	6808
370	56820	56832	56844	56855	56867	56879	56891	56902	56914	56926
1	6937	6949	6961	6972	6984	6996	7008	7019	7031	7043
2	7054	7066	7078	7089	7101	7113	7124	7136	7148	7159
3	7171	7183	7194	7206	7217	7229	7241	7252	7264	7276
4	7287	7299	7310	7322	7334	7345	7357	7368	7380	7392
5	7403	7415	7426	7438	7449	7461	7473	7484	7496	7507
6	7519	7530	7542	7553	7565	7576	7588	7600	7611	7623
7	7634	7646	7657	7669	7680	7692	7703	7715	7726	7738
8	7749	7761	7772	7784	7795	7807	7818	7830	7841	7852
9	7864	7875	7887	7898	7910	7921	7933	7944	7955	7967
380	57978	57990	58001	58013	58024	58035	58047	58058	58070	58081
1	8092	8104	8115	8127	8138	8149	8161	8172	8184	8195
2	8206	8218	8229	8240	8252	8263	8274	8286	8297	8309
3	8320	8331	8343	8354	8365	8377	8388	8399	8410	8422
4	8433	8444	8456	8467	8478	8490	8501	8512	8524	8535
5	8546	8557	8569	8580	8591	8602	8614	8625	8636	8647
6	8659	8670	8681	8692	8704	8715	8726	8737	8749	8760
7	8771	8782	8794	8805	8816	8827	8838	8850	8861	8872
8	8883	8894	8906	8917	8928	8939	8950	8961	8973	8984
9	8995	9006	9017	9028	9040	9051	9062	9073	9084	9095
390	59106	59118	59129	59140	59151	59162	59173	59184	59195	59207
1	9218	9229	9240	9251	9262	9273	9284	9295	9306	9318
2	9329	9340	9351	9362	9373	9384	9395	9406	9417	9428
3	9439	9450	9461	9472	9483	9494	9506	9517	9528	9539
4	9550	9561	9572	9583	9594	9605	9616	9627	9638	9649
5	9660	9671	9682	9693	9704	9715	9726	9737	9748	9759
6	9770	9780	9791	9802	9813	9824	9835	9846	9857	9868
7	9879	9890	9901	9912	9923	9934	9945	9956	9966	9977
8	9988	9999	60010	60021	60032	60043	60054	60065	60076	60086
9	60097	60108	0119	0130	0141	0152	0163	0173	0184	0195
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
400	60206	60217	60228	60239	60249	60260	60271	60282	60293	60304
1	0314	0325	0336	0347	0358	0369	0379	0390	0401	0412
2	0423	0433	0444	0455	0466	0477	0487	0498	0509	0520
3	0531	0541	0552	0563	0574	0584	0595	0606	0617	0627
4	0638	0649	0660	0670	0681	0692	0703	0713	0724	0735
5	0746	0756	0767	0778	0788	0799	0810	0821	0831	0842
6	0853	0863	0874	0885	0895	0906	0917	0927	0938	0949
7	0959	0970	0981	0991	1002	1013	1023	1034	1045	1055
8	1066	1077	1087	1098	1109	1119	1130	1140	1151	1162
9	1172	1183	1194	1204	1215	1225	1236	1247	1257	1268
410	61278	61289	61300	61310	61321	61331	61342	61352	61363	61374
1	1384	1395	1405	1416	1426	1437	1448	1458	1469	1479
2	1490	1500	1511	1521	1532	1542	1553	1563	1574	1584
3	1595	1606	1616	1627	1637	1648	1658	1669	1679	1690
4	1700	1711	1721	1731	1742	1752	1763	1773	1784	1794
5	1805	1815	1826	1836	1847	1857	1868	1878	1888	1899
6	1909	1920	1930	1941	1951	1962	1972	1982	1993	2003
7	2014	2024	2034	2045	2055	2066	2076	2086	2097	2107
8	2118	2128	2138	2149	2159	2170	2180	2190	2201	2211
9	2221	2232	2242	2252	2263	2273	2284	2294	2304	2315
420	62325	62335	62346	62356	62366	62377	62387	62397	62408	62418
1	2428	2439	2449	2459	2469	2480	2490	2500	2511	2521
2	2531	2542	2552	2562	2572	2583	2593	2603	2613	2624
3	2634	2644	2655	2665	2675	2685	2696	2706	2716	2726
4	2737	2747	2757	2767	2778	2788	2798	2808	2818	2829
5	2839	2849	2859	2870	2880	2890	2900	2910	2921	2931
6	2941	2951	2961	2972	2982	2992	3002	3012	3022	3033
7	3043	3053	3063	3073	3083	3094	3104	3114	3124	3134
8	3144	3155	3165	3175	3185	3195	3205	3215	3225	3236
9	3246	3256	3266	3276	3286	3296	3306	3317	3327	3337
430	63347	63357	63367	63377	63387	63397	63407	63417	63428	63438
1	3448	3458	3468	3478	3488	3498	3508	3518	3528	3538
2	3548	3558	3568	3579	3589	3599	3609	3619	3629	3639
3	3649	3659	3669	3679	3689	3699	3709	3719	3729	3739
4	3749	3759	3769	3779	3789	3799	3809	3819	3829	3839
5	3849	3859	3869	3879	3889	3899	3909	3919	3929	3939
6	3949	3959	3969	3979	3988	3998	4008	4018	4028	4038
7	4048	4058	4068	4078	4088	4098	4108	4118	4128	4137
8	4147	4157	4167	4177	4187	4197	4207	4217	4227	4237
9	4246	4256	4266	4276	4286	4296	4306	4316	4326	4335
440	64345	64355	64365	64375	64385	64395	64404	64414	64424	64434
1	4444	4454	4464	4473	4483	4493	4503	4513	4523	4532
2	4542	4552	4562	4572	4582	4591	4601	4611	4621	4631
3	4640	4650	4660	4670	4680	4689	4699	4709	4719	4729
4	4738	4748	4758	4768	4777	4787	4797	4807	4816	4826
5	4836	4846	4856	4865	4875	4885	4895	4904	4914	4924
6	4933	4943	4953	4963	4972	4982	4992	5002	5011	5021
7	5031	5040	5050	5060	5070	5079	5089	5099	5108	5118
8	5128	5137	5147	5157	5167	5176	5186	5196	5205	5215
9	5225	5234	5244	5254	5263	5273	5283	5292	5302	5312
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
450	65321	65331	65341	65350	65360	65369	65379	65389	65398	65408
1	5418	5427	5437	5447	5456	5466	5475	5485	5495	5504
2	5514	5523	5533	5543	5552	5562	5571	5581	5591	5600
3	5610	5619	5629	5639	5648	5658	5667	5677	5686	5696
4	5706	5715	5725	5734	5744	5753	5763	5772	5782	5792
5	5801	5811	5820	5830	5839	5849	5858	5868	5877	5887
6	5896	5906	5916	5925	5935	5944	5954	5963	5973	5982
7	5992	6001	6011	6020	6030	6039	6049	6058	6068	6077
8	6087	6096	6106	6115	6124	6134	6143	6153	6162	6172
9	6181	6191	6200	6210	6219	6229	6238	6247	6257	6266
460	66276	66285	66295	66304	66314	66323	66332	66342	66351	66361
1	6370	6380	6389	6398	6408	6417	6427	6436	6445	6455
2	6464	6474	6483	6492	6502	6511	6521	6530	6539	6549
3	6558	6567	6577	6586	6596	6605	6614	6624	6633	6642
4	6652	6661	6671	6680	6689	6699	6708	6717	6727	6736
5	6745	6755	6764	6773	6783	6792	6801	6811	6820	6829
6	6839	6848	6857	6867	6876	6885	6894	6904	6913	6922
7	6932	6941	6950	6960	6969	6978	6987	6997	7006	7015
8	7025	7034	7043	7052	7062	7071	7080	7089	7099	7108
9	7117	7127	7136	7145	7154	7164	7173	7182	7191	7201
470	67210	67219	67228	67237	67247	67256	67265	67274	67284	67293
1	7302	7311	7321	7330	7339	7348	7357	7367	7376	7385
2	7394	7403	7413	7422	7431	7440	7449	7459	7468	7477
3	7486	7495	7504	7514	7523	7532	7541	7550	7560	7569
4	7578	7587	7596	7605	7614	7624	7633	7642	7651	7660
5	7669	7679	7688	7697	7706	7715	7724	7733	7742	7752
6	7761	7770	7779	7788	7797	7806	7815	7825	7834	7843
7	7852	7861	7870	7879	7888	7897	7906	7916	7925	7934
8	7943	7952	7961	7970	7979	7988	7997	8006	8015	8024
9	8034	8043	8052	8061	8070	8079	8088	8097	8106	8115
480	68124	68133	68142	68151	68160	68169	68178	68187	68196	68205
1	8215	8224	8233	8242	8251	8260	8269	8278	8287	8296
2	8305	8314	8323	8332	8341	8350	8359	8368	8377	8386
3	8395	8404	8413	8422	8431	8440	8449	8458	8467	8476
4	8485	8494	8502	8511	8520	8529	8538	8547	8556	8565
5	8574	8583	8592	8601	8610	8619	8628	8637	8646	8655
6	8664	8673	8681	8690	8699	8708	8717	8726	8735	8744
7	8753	8762	8771	8780	8789	8797	8806	8815	8824	8833
8	8842	8851	8860	8869	8878	8886	8895	8904	8913	8922
9	8931	8940	8949	8958	8966	8975	8984	8993	9002	9011
490	69020	69028	69037	69046	69055	69064	69073	69082	69090	69099
1	9108	9117	9126	9135	9144	9152	9161	9170	9179	9188
2	9197	9205	9214	9223	9232	9241	9249	9258	9267	9276
3	9285	9294	9302	9311	9320	9329	9338	9346	9355	9364
4	9373	9381	9390	9399	9408	9417	9425	9434	9443	9452
5	9461	9469	9478	9487	9496	9504	9513	9522	9531	9539
6	9548	9557	9566	9574	9583	9592	9601	9609	9618	9627
7	9636	9644	9653	9662	9671	9679	9688	9697	9705	9714
8	9723	9732	9740	9749	9758	9767	9775	9784	9793	9801
9	9810	9819	9827	9836	9845	9854	9862	9871	9880	9888
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
500	69897	69906	69914	69923	69932	69940	69949	69958	69966	69975
1	9984	9992	70001	70010	70018	70027	70036	70044	70053	70062
2	70070	70079	0088	0096	0105	0114	0122	0131	0140	0148
3	0157	0165	0174	0183	0191	0200	0209	0217	0226	0234
4	0243	0252	0260	0269	0278	0286	0295	0303	0312	0321
5	0329	0338	0346	0355	0364	0372	0381	0389	0398	0406
6	0415	0424	0432	0441	0449	0458	0467	0475	0484	0492
7	0501	0509	0518	0526	0535	0544	0552	0561	0569	0578
8	0586	0595	0603	0612	0621	0629	0638	0646	0655	0663
9	0672	0680	0689	0697	0706	0714	0723	0731	0740	0749
510	70757	70766	70774	70783	70791	70800	70808	70817	70825	70834
1	0842	0851	0859	0868	0876	0885	0893	0902	0910	0919
2	0927	0935	0944	0952	0961	0969	0978	0986	0995	1003
3	1012	1020	1029	1037	1046	1054	1063	1071	1079	1088
4	1096	1105	1113	1122	1130	1139	1147	1155	1164	1172
5	1181	1189	1198	1206	1214	1223	1231	1240	1248	1257
6	1265	1273	1282	1290	1299	1307	1315	1324	1332	1341
7	1349	1357	1366	1374	1383	1391	1399	1408	1416	1425
8	1433	1441	1450	1458	1466	1475	1483	1492	1500	1508
9	1517	1525	1533	1542	1550	1559	1567	1575	1584	1592
520	71600	71609	71617	71625	71634	71642	71650	71659	71667	71675
1	1684	1692	1700	1709	1717	1725	1734	1742	1750	1759
2	1767	1775	1784	1792	1800	1809	1817	1825	1834	1842
3	1850	1858	1867	1875	1883	1892	1900	1908	1917	1925
4	1933	1941	1950	1958	1966	1975	1983	1991	1999	2008
5	2016	2024	2032	2041	2049	2057	2066	2074	2082	2090
6	2099	2107	2115	2123	2132	2140	2148	2156	2165	2173
7	2181	2189	2198	2206	2214	2222	2230	2239	2247	2255
8	2263	2272	2280	2288	2296	2304	2313	2321	2329	2337
9	2346	2354	2362	2370	2378	2387	2395	2403	2411	2419
530	72428	72436	72444	72452	72460	72469	72477	72485	72493	72501
1	2509	2518	2526	2534	2542	2550	2558	2567	2575	2583
2	2591	2599	2607	2616	2624	2632	2640	2648	2656	2665
3	2673	2681	2689	2697	2705	2713	2722	2730	2738	2746
4	2754	2762	2770	2779	2787	2795	2803	2811	2819	2827
5	2835	2843	2852	2860	2868	2876	2884	2892	2900	2908
6	2916	2925	2933	2941	2949	2957	2965	2973	2981	2989
7	2997	3006	3014	3022	3030	3038	3046	3054	3062	3070
8	3078	3086	3094	3102	3111	3119	3127	3135	3143	3151
9	3159	3167	3175	3183	3191	3199	3207	3215	3223	3231
540	73239	73247	73255	73263	73272	73280	73288	73296	73304	73312
1	3320	3328	3336	3344	3352	3360	3368	3376	3384	3392
2	3400	3408	3416	3424	3432	3440	3448	3456	3464	3472
3	3480	3488	3496	3504	3512	3520	3528	3536	3544	3552
4	3560	3568	3576	3584	3592	3600	3608	3616	3624	3632
5	3640	3648	3656	3664	3672	3679	3687	3695	3703	3711
6	3719	3727	3735	3743	3751	3759	3767	3775	3783	3791
7	3799	3807	3815	3823	3830	3838	3846	3854	3862	3870
8	3878	3886	3894	3902	3910	3918	3926	3933	3941	3949
9	3957	3965	3973	3981	3989	3997	4005	4013	4020	4028
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
550	74036	74044	74052	74060	74068	74076	74084	74092	74099	74107
1	4115	4123	4131	4139	4147	4155	4162	4170	4178	4186
2	4194	4202	4210	4218	4225	4233	4241	4249	4257	4265
3	4273	4280	4288	4296	4304	4312	4320	4327	4335	4343
4	4351	4359	4367	4374	4382	4390	4398	4406	4414	4421
5	4429	4437	4445	4453	4461	4468	4476	4484	4492	4500
6	4507	4515	4523	4531	4539	4547	4554	4562	4570	4578
7	4586	4593	4601	4609	4617	4624	4632	4640	4648	4656
8	4663	4671	4679	4687	4695	4702	4710	4718	4726	4733
9	4741	4749	4757	4764	4772	4780	4788	4796	4803	4811
560	74819	74827	74834	74842	74850	74858	74865	74873	74881	74889
1	4896	4904	4912	4920	4927	4935	4943	4950	4958	4966
2	4974	4981	4989	4997	5005	5012	5020	5028	5035	5043
3	5051	5059	5066	5074	5082	5089	5097	5105	5113	5120
4	5128	5136	5143	5151	5159	5166	5174	5182	5189	5197
5	5205	5213	5220	5228	5236	5243	5251	5259	5266	5274
6	5282	5289	5297	5305	5312	5320	5328	5335	5343	5351
7	5358	5366	5374	5381	5389	5397	5404	5412	5420	5427
8	5435	5442	5450	5458	5465	5473	5481	5488	5496	5504
9	5511	5519	5526	5534	5542	5549	5557	5565	5572	5580
570	75587	75595	75603	75610	75618	75626	75633	75641	75648	75656
1	5664	5671	5679	5686	5694	5702	5709	5717	5724	5732
2	5740	5747	5755	5762	5770	5778	5785	5793	5800	5808
3	5815	5823	5831	5838	5846	5853	5861	5868	5876	5884
4	5891	5899	5906	5914	5921	5929	5937	5944	5952	5959
5	5967	5974	5982	5989	5997	6005	6012	6020	6027	6035
6	6042	6050	6057	6065	6072	6080	6087	6095	6103	6110
7	6118	6125	6133	6140	6148	6155	6163	6170	6178	6185
8	6193	6200	6208	6215	6223	6230	6238	6245	6253	6260
9	6268	6275	6283	6290	6298	6305	6313	6320	6328	6335
580	76343	76350	76358	76365	76373	76380	76388	76395	76403	76410
1	6418	6425	6433	6440	6448	6455	6462	6470	6477	6485
2	6492	6500	6507	6515	6522	6530	6537	6545	6552	6559
3	6567	6574	6582	6589	6597	6604	6612	6619	6626	6634
4	6641	6649	6656	6664	6671	6678	6686	6693	6701	6708
5	6716	6723	6730	6738	6745	6753	6760	6768	6775	6782
6	6790	6797	6805	6812	6819	6827	6834	6842	6849	6856
7	6864	6871	6879	6886	6893	6901	6908	6916	6923	6930
8	6938	6945	6953	6960	6967	6975	6982	6989	6997	7004
9	7012	7019	7026	7034	7041	7048	7056	7063	7070	7078
590	77085	77093	77100	77107	77115	77122	77129	77137	77144	77151
1	7159	7166	7173	7181	7188	7195	7203	7210	7217	7225
2	7232	7240	7247	7254	7262	7269	7276	7283	7291	7298
3	7305	7313	7320	7327	7335	7342	7349	7357	7364	7371
4	7379	7386	7393	7401	7408	7415	7422	7430	7437	7444
5	7452	7459	7466	7474	7481	7488	7495	7503	7510	7517
6	7525	7532	7539	7546	7554	7561	7568	7576	7583	7590
7	7597	7605	7612	7619	7627	7634	7641	7648	7656	7663
8	7670	7677	7685	7692	7699	7706	7714	7721	7728	7735
9	7743	7750	7757	7764	7772	7779	7786	7793	7801	7808
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
600	77815	77822	77830	77837	77844	77851	77859	77866	77873	77880
1	7887	7895	7902	7909	7916	7924	7931	7938	7945	7952
2	7960	7967	7974	7981	7988	7996	8003	8010	8017	8025
3	8032	8039	8046	8053	8061	8068	8075	8082	8089	8097
4	8104	8111	8118	8125	8132	8140	8147	8154	8161	8168
5	8176	8183	8190	8197	8204	8211	8219	8226	8233	8240
6	8247	8254	8262	8269	8276	8283	8290	8297	8305	8312
7	8319	8326	8333	8340	8347	8355	8362	8369	8376	8383
8	8390	8398	8405	8412	8419	8426	8433	8440	8447	8455
9	8462	8469	8476	8483	8490	8497	8504	8512	8519	8526
610	78533	78540	78547	78554	78561	78569	78576	78583	78590	78597
1	8604	8611	8618	8625	8633	8640	8647	8654	8661	8668
2	8675	8682	8689	8696	8704	8711	8718	8725	8732	8739
3	8746	8753	8760	8767	8774	8781	8789	8796	8803	8810
4	8817	8824	8831	8838	8845	8852	8859	8866	8873	8880
5	8888	8895	8902	8909	8916	8923	8930	8937	8944	8951
6	8958	8965	8972	8979	8986	8993	9000	9007	9014	9021
7	9029	9036	9043	9050	9057	9064	9071	9078	9085	9092
8	9099	9106	9113	9120	9127	9134	9141	9148	9155	9162
9	9169	9176	9183	9190	9197	9204	9211	9218	9225	9232
620	79239	79246	79253	79260	79267	79274	79281	79288	79295	79302
1	9309	9316	9323	9330	9337	9344	9351	9358	9365	9372
2	9379	9386	9393	9400	9407	9414	9421	9428	9435	9442
3	9449	9456	9463	9470	9477	9484	9491	9498	9505	9511
4	9518	9525	9532	9539	9546	9553	9560	9567	9574	9581
5	9588	9595	9602	9609	9616	9623	9630	9637	9644	9650
6	9657	9664	9671	9678	9685	9692	9699	9706	9713	9720
7	9727	9734	9741	9748	9754	9761	9768	9775	9782	9789
8	9796	9803	9810	9817	9824	9831	9837	9844	9851	9858
9	9865	9872	9879	9886	9893	9900	9906	9913	9920	9927
630	79934	79941	79948	79955	79962	79969	79975	79982	79989	79996
1	80003	80010	80017	80024	80030	80037	80044	80051	80058	80065
2	0072	0079	0085	0092	0099	0106	0113	0120	0127	0134
3	0140	0147	0154	0161	0168	0175	0182	0188	0195	0202
4	0209	0216	0223	0229	0236	0243	0250	0257	0264	0271
5	0277	0284	0291	0298	0305	0312	0318	0325	0332	0339
6	0346	0353	0359	0366	0373	0380	0387	0393	0400	0407
7	0414	0421	0428	0434	0441	0448	0455	0462	0468	0475
8	0482	0489	0496	0502	0509	0516	0523	0530	0536	0543
9	0550	0557	0564	0570	0577	0584	0591	0598	0604	0611
640	80618	80625	80632	80638	80645	80652	80659	80665	80672	80679
1	0686	0693	0699	0706	0713	0720	0726	0733	0740	0747
2	0754	0760	0767	0774	0781	0787	0794	0801	0808	0814
3	0821	0828	0835	0841	0848	0855	0862	0868	0875	0882
4	0889	0895	0902	0909	0916	0922	0929	0936	0943	0949
5	0956	0963	0969	0976	0983	0990	0996	1003	1010	1017
6	1023	1030	1037	1043	1050	1057	1064	1070	1077	1084
7	1090	1097	1104	1111	1117	1124	1131	1137	1144	1151
8	1158	1164	1171	1178	1184	1191	1198	1204	1211	1218
9	1224	1231	1238	1245	1251	1258	1265	1271	1278	1285
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
650	81291	81298	81305	81311	81318	81325	81331	81338	81345	81351
1	1358	1365	1371	1378	1385	1391	1398	1405	1411	1418
2	1425	1431	1438	1445	1451	1458	1465	1471	1478	1485
3	1491	1498	1505	1511	1518	1525	1531	1538	1544	1551
4	1558	1564	1571	1578	1584	1591	1598	1604	1611	1617
5	1624	1631	1637	1644	1651	1657	1664	1671	1677	1684
6	1690	1697	1704	1710	1717	1723	1730	1737	1743	1750
7	1757	1763	1770	1776	1783	1790	1796	1803	1809	1816
8	1823	1829	1836	1842	1849	1856	1862	1869	1875	1882
9	1889	1895	1902	1908	1915	1921	1928	1935	1941	1948
660	81954	81961	81968	81974	81981	81987	81994	82000	82007	82014
1	2020	2027	2033	2040	2046	2053	2060	2066	2073	2079
2	2086	2092	2099	2105	2112	2119	2125	2132	2138	2145
3	2151	2158	2164	2171	2178	2184	2191	2197	2204	2210
4	2217	2223	2230	2236	2243	2249	2256	2263	2269	2276
5	2282	2289	2295	2302	2308	2315	2321	2328	2334	2341
6	2347	2354	2360	2367	2373	2380	2387	2393	2400	2406
7	2413	2419	2426	2432	2439	2445	2452	2458	2465	2471
8	2478	2484	2491	2497	2504	2510	2517	2523	2530	2536
9	2543	2549	2556	2562	2569	2575	2582	2588	2595	2601
670	82607	82614	82620	82627	82633	82640	82646	82653	82659	82666
1	2672	2679	2685	2692	2698	2705	2711	2718	2724	2730
2	2737	2743	2750	2756	2763	2769	2776	2782	2789	2795
3	2802	2808	2814	2821	2827	2834	2840	2847	2853	2860
4	2866	2872	2879	2885	2892	2898	2905	2911	2918	2924
5	2930	2937	2943	2950	2956	2963	2969	2975	2982	2988
6	2995	3001	3008	3014	3020	3027	3033	3040	3046	3052
7	3059	3065	3072	3078	3085	3091	3097	3104	3110	3117
8	3123	3129	3136	3142	3149	3155	3161	3168	3174	3181
9	3187	3193	3200	3206	3213	3219	3225	3232	3238	3245
680	83251	83257	83264	83270	83276	83283	83289	83296	83302	83308
1	3315	3321	3327	3334	3340	3347	3353	3359	3366	3372
2	3378	3385	3391	3398	3404	3410	3417	3423	3429	3436
3	3442	3448	3455	3461	3467	3474	3480	3487	3493	3499
4	3506	3512	3518	3525	3531	3537	3544	3550	3556	3563
5	3569	3575	3582	3588	3594	3601	3607	3613	3620	3626
6	3632	3639	3645	3651	3658	3664	3670	3677	3683	3689
7	3696	3702	3708	3715	3721	3727	3734	3740	3746	3753
8	3759	3765	3771	3778	3784	3790	3797	3803	3809	3816
9	3822	3828	3835	3841	3847	3853	3860	3866	3872	3879
690	83885	83891	83897	83904	83910	83916	83923	83929	83935	83942
1	3948	3954	3960	3967	3973	3979	3985	3992	3998	4004
2	4011	4017	4023	4029	4036	4042	4048	4055	4061	4067
3	4073	4080	4086	4092	4098	4105	4111	4117	4123	4130
4	4136	4142	4148	4155	4161	4167	4173	4180	4186	4192
5	4198	4205	4211	4217	4223	4230	4236	4242	4248	4255
6	4261	4267	4273	4280	4286	4292	4298	4305	4311	4317
7	4323	4330	4336	4342	4348	4354	4361	4367	4373	4379
8	4386	4392	4398	4404	4410	4417	4423	4429	4435	4442
9	4448	4454	4460	4466	4473	4479	4485	4491	4497	4504
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
700	84510	84516	84522	84528	84535	84541	84547	84553	84559	84566
1	4572	4578	4584	4590	4597	4603	4609	4615	4621	4628
2	4634	4640	4646	4652	4658	4665	4671	4677	4683	4689
3	4696	4702	4708	4714	4720	4726	4733	4739	4745	4751
4	4757	4763	4770	4776	4782	4788	4794	4800	4807	4813
5	4819	4825	4831	4837	4844	4850	4856	4862	4868	4874
6	4880	4887	4893	4899	4905	4911	4917	4924	4930	4936
7	4942	4948	4954	4960	4967	4973	4979	4985	4991	4997
8	5003	5009	5016	5022	5028	5034	5040	5046	5052	5058
9	5065	5071	5077	5083	5089	5095	5101	5107	5114	5120
710	85126	85132	85138	85144	85150	85156	85163	85169	85175	85181
1	5187	5193	5199	5205	5211	5217	5224	5230	5236	5242
2	5248	5254	5260	5266	5272	5278	5285	5291	5297	5303
3	5309	5315	5321	5327	5333	5339	5345	5352	5358	5364
4	5370	5376	5382	5388	5394	5400	5406	5412	5418	5425
5	5431	5437	5443	5449	5455	5461	5467	5473	5479	5485
6	5491	5497	5503	5509	5516	5522	5528	5534	5540	5546
7	5552	5558	5564	5570	5576	5582	5588	5594	5600	5606
8	5612	5618	5625	5631	5637	5643	5649	5655	5661	5667
9	5673	5679	5685	5691	5697	5703	5709	5715	5721	5727
720	85733	85739	85745	85751	85757	85763	85769	85775	85781	85788
1	5794	5800	5806	5812	5818	5824	5830	5836	5842	5848
2	5854	5860	5866	5872	5878	5884	5890	5896	5902	5908
3	5914	5920	5926	5932	5938	5944	5950	5956	5962	5968
4	5974	5980	5986	5992	5998	6004	6010	6016	6022	6028
5	6034	6040	6046	6052	6058	6064	6070	6076	6082	6088
6	6094	6100	6106	6112	6118	6124	6130	6136	6141	6147
7	6153	6159	6165	6171	6177	6183	6189	6195	6201	6207
8	6213	6219	6225	6231	6237	6243	6249	6255	6261	6267
9	6273	6279	6285	6291	6297	6303	6308	6314	6320	6326
730	86332	86338	86344	86350	86356	86362	86368	86374	86380	86386
1	6392	6398	6404	6410	6415	6421	6427	6433	6439	6445
2	6451	6457	6463	6469	6475	6481	6487	6493	6499	6504
3	6510	6516	6522	6528	6534	6540	6546	6552	6558	6564
4	6570	6576	6581	6587	6593	6599	6605	6611	6617	6623
5	6629	6635	6641	6646	6652	6658	6664	6670	6676	6682
6	6688	6694	6700	6705	6711	6717	6723	6729	6735	6741
7	6747	6753	6759	6764	6770	6776	6782	6788	6794	6800
8	6806	6812	6817	6823	6829	6835	6841	6847	6853	6859
9	6864	6870	6876	6882	6888	6894	6900	6906	6911	6917
740	86923	86929	86935	86941	86947	86953	86958	86964	86970	86976
1	6982	6988	6994	6999	7005	7011	7017	7023	7029	7035
2	7040	7046	7052	7058	7064	7070	7075	7081	7087	7093
3	7099	7105	7111	7116	7122	7128	7134	7140	7146	7151
4	7157	7163	7169	7175	7181	7186	7192	7198	7204	7210
5	7216	7221	7227	7233	7239	7245	7251	7256	7262	7268
6	7274	7280	7286	7291	7297	7303	7309	7315	7320	7326
7	7332	7338	7344	7349	7355	7361	7367	7373	7379	7384
8	7390	7396	7402	7408	7413	7419	7425	7431	7437	7442
9	7448	7454	7460	7466	7471	7477	7483	7489	7495	7500
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
750	87506	87512	87518	87523	87529	87535	87541	87547	87552	87558
1	7564	7570	7576	7581	7587	7593	7599	7604	7610	7616
2	7622	7628	7633	7639	7645	7651	7656	7662	7668	7674
3	7679	7685	7691	7697	7703	7708	7714	7720	7726	7731
4	7737	7743	7749	7754	7760	7766	7772	7777	7783	7789
5	7795	7800	7806	7812	7818	7823	7829	7835	7841	7846
6	7852	7858	7864	7869	7875	7881	7887	7892	7898	7904
7	7910	7915	7921	7927	7933	7938	7944	7950	7955	7961
8	7967	7973	7978	7984	7990	7996	8001	8007	8013	8018
9	8024	8030	8036	8041	8047	8053	8058	8064	8070	8076
760	88081	88087	88093	88098	88104	88110	88116	88121	88127	88133
1	8138	8144	8150	8156	8161	8167	8173	8178	8184	8190
2	8195	8201	8207	8213	8218	8224	8230	8235	8241	8247
3	8252	8258	8264	8270	8275	8281	8287	8292	8298	8304
4	8309	8315	8321	8326	8332	8338	8343	8349	8355	8360
5	8366	8372	8377	8383	8389	8395	8400	8406	8412	8417
6	8423	8429	8434	8440	8446	8451	8457	8463	8468	8474
7	8480	8485	8491	8497	8502	8508	8513	8519	8525	8530
8	8536	8542	8547	8553	8559	8564	8570	8576	8581	8587
9	8593	8598	8604	8610	8615	8621	8627	8632	8638	8643
770	88649	88655	88660	88666	88672	88677	88683	88689	88694	88700
1	8705	8711	8717	8722	8728	8734	8739	8745	8750	8756
2	8762	8767	8773	8779	8784	8790	8795	8801	8807	8812
3	8818	8824	8829	8835	8840	8846	8852	8857	8863	8868
4	8874	8880	8885	8891	8897	8902	8908	8913	8919	8925
5	8930	8936	8941	8947	8953	8958	8964	8969	8975	8981
6	8986	8992	8997	9003	9009	9014	9020	9025	9031	9037
7	9042	9048	9053	9059	9064	9070	9076	9081	9087	9092
8	9098	9104	9109	9115	9120	9126	9131	9137	9143	9148
9	9154	9159	9165	9170	9176	9182	9187	9193	9198	9204
780	89209	89215	89221	89226	89232	89237	89243	89248	89254	89260
1	9265	9271	9276	9282	9287	9293	9298	9304	9310	9315
2	9321	9326	9332	9337	9343	9348	9354	9360	9365	9371
3	9376	9382	9387	9393	9398	9404	9409	9415	9421	9426
4	9432	9437	9443	9448	9454	9459	9465	9470	9476	9481
5	9487	9492	9498	9504	9509	9515	9520	9526	9531	9537
6	9542	9548	9553	9559	9564	9570	9575	9581	9586	9592
7	9597	9603	9609	9614	9620	9625	9631	9636	9642	9647
8	9653	9658	9664	9669	9675	9680	9686	9691	9697	9702
9	9708	9713	9719	9724	9730	9735	9741	9746	9752	9757
790	89763	89768	89774	89779	89785	89790	89796	89801	89807	89812
1	9818	9823	9829	9834	9840	9845	9851	9856	9862	9867
2	9873	9878	9883	9889	9894	9900	9905	9911	9916	9922
3	9927	9933	9938	9944	9949	9955	9960	9966	9971	9977
4	9982	9988	9993	9998	90004	90009	90015	90020	90026	90031
5	90037	90042	90048	90053	0059	0064	0069	0075	0080	0086
6	0091	0097	0102	0108	0113	0119	0124	0129	0135	0140
7	0146	0151	0157	0162	0168	0173	0179	0184	0189	0195
8	0200	0206	0211	0217	0222	0227	0233	0238	0244	0249
9	0255	0260	0266	0271	0276	0282	0287	0293	0298	0304
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
800	90309	90314	90320	90325	90331	90336	90342	90347	90352	90358
1	0363	0369	0374	0380	0385	0390	0396	0401	0407	0412
2	0417	0423	0428	0434	0439	0445	0450	0455	0461	0466
3	0472	0477	0482	0488	0493	0499	0504	0509	0515	0520
4	0526	0531	0536	0542	0547	0553	0558	0563	0569	0574
5	0580	0585	0590	0596	0601	0607	0612	0617	0623	0628
6	0634	0639	0644	0650	0655	0660	0666	0671	0677	0682
7	0687	0693	0698	0703	0709	0714	0720	0725	0730	0736
8	0741	0747	0752	0757	0763	0768	0773	0779	0784	0789
9	0795	0800	0806	0811	0816	0822	0827	0832	0838	0843
810	90849	90854	90859	90865	90870	90875	90881	90886	90891	90897
1	0902	0907	0913	0918	0924	0929	0934	0940	0945	0950
2	0956	0961	0966	0972	0977	0982	0988	0993	0998	1004
3	1009	1014	1020	1025	1030	1036	1041	1046	1052	1057
4	1062	1068	1073	1078	1084	1089	1094	1100	1105	1110
5	1116	1121	1126	1132	1137	1142	1148	1153	1158	1164
6	1169	1174	1180	1185	1190	1196	1201	1206	1212	1217
7	1222	1228	1233	1238	1243	1249	1254	1259	1265	1270
8	1275	1281	1286	1291	1297	1302	1307	1312	1318	1323
9	1328	1334	1339	1344	1350	1355	1360	1365	1371	1376
820	91381	91387	91392	91397	91403	91408	91413	91418	91424	91429
1	1434	1440	1445	1450	1455	1461	1466	1471	1477	1482
2	1487	1492	1498	1503	1508	1514	1519	1524	1529	1535
3	1540	1545	1551	1556	1561	1566	1572	1577	1582	1587
4	1593	1598	1603	1609	1614	1619	1624	1630	1635	1640
5	1645	1651	1656	1661	1666	1672	1677	1682	1687	1693
6	1698	1703	1709	1714	1719	1724	1730	1735	1740	1745
7	1751	1756	1761	1766	1772	1777	1782	1787	1793	1798
8	1803	1808	1814	1819	1824	1829	1834	1840	1845	1850
9	1855	1861	1866	1871	1876	1882	1887	1892	1897	1903
830	91908	91913	91918	91924	91929	91934	91939	91944	91950	91955
1	1960	1965	1971	1976	1981	1986	1991	1997	2002	2007
2	2012	2018	2023	2028	2033	2038	2044	2049	2054	2059
3	2065	2070	2075	2080	2085	2091	2096	2101	2106	2111
4	2117	2122	2127	2132	2137	2143	2148	2153	2158	2163
5	2169	2174	2179	2184	2189	2195	2200	2205	2210	2215
6	2221	2226	2231	2236	2241	2247	2252	2257	2262	2267
7	2273	2278	2283	2288	2293	2298	2304	2309	2314	2319
8	2324	2330	2335	2340	2345	2350	2355	2361	2366	2371
9	2376	2381	2387	2392	2397	2402	2407	2412	2418	2423
840	92428	92433	92438	92443	92449	92454	92459	92464	92469	92474
1	2480	2485	2490	2495	2500	2505	2511	2516	2521	2526
2	2531	2536	2542	2547	2552	2557	2562	2567	2572	2578
3	2583	2588	2593	2598	2603	2609	2614	2619	2624	2629
4	2634	2639	2645	2650	2655	2660	2665	2670	2675	2681
5	2686	2691	2696	2701	2706	2711	2716	2722	2727	2732
6	2737	2742	2747	2752	2758	2763	2768	2773	2778	2783
7	2788	2793	2799	2804	2809	2814	2819	2824	2829	2834
8	2840	2845	2850	2855	2860	2865	2870	2875	2881	2886
9	2891	2896	2901	2906	2911	2916	2921	2927	2932	2937
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988
1	2993	2998	3003	3008	3013	3018	3024	3029	3034	3039
2	3044	3049	3054	3059	3064	3069	3075	3080	3085	3090
3	3095	3100	3105	3110	3115	3120	3125	3131	3136	3141
4	3146	3151	3156	3161	3166	3171	3176	3181	3186	3192
5	3197	3202	3207	3212	3217	3222	3227	3232	3237	3242
6	3247	3252	3258	3263	3268	3273	3278	3283	3288	3293
7	3298	3303	3308	3313	3318	3323	3328	3334	3339	3344
8	3349	3354	3359	3364	3369	3374	3379	3384	3389	3394
9	3399	3404	3409	3414	3420	3425	3430	3435	3440	3445
860	93450	93455	93460	93465	93470	93475	93480	93485	93490	93495
1	3500	3505	3510	3515	3520	3526	3531	3536	3541	3546
2	3551	3556	3561	3566	3571	3576	3581	3586	3591	3596
3	3601	3606	3611	3616	3621	3626	3631	3636	3641	3646
4	3651	3656	3661	3666	3671	3676	3682	3687	3692	3697
5	3702	3707	3712	3717	3722	3727	3732	3737	3742	3747
6	3752	3757	3762	3767	3772	3777	3782	3787	3792	3797
7	3802	3807	3812	3817	3822	3827	3832	3837	3842	3847
8	3852	3857	3862	3867	3872	3877	3882	3887	3892	3897
9	3902	3907	3912	3917	3922	3927	3932	3937	3942	3947
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997
1	4002	4007	4012	4017	4022	4027	4032	4037	4042	4047
2	4052	4057	4062	4067	4072	4077	4082	4086	4091	4096
3	4101	4106	4111	4116	4121	4126	4131	4136	4141	4146
4	4151	4156	4161	4166	4171	4176	4181	4186	4191	4196
5	4201	4206	4211	4216	4221	4226	4231	4236	4240	4245
6	4250	4255	4260	4265	4270	4275	4280	4285	4290	4295
7	4300	4305	4310	4315	4320	4325	4330	4335	4340	4345
8	4349	4354	4359	4364	4369	4374	4379	4384	4389	4394
9	4399	4404	4409	4414	4419	4424	4429	4433	4438	4443
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493
1	4498	4503	4507	4512	4517	4522	4527	4532	4537	4542
2	4547	4552	4557	4562	4567	4571	4576	4581	4586	4591
3	4596	4601	4606	4611	4616	4621	4626	4630	4635	4640
4	4645	4650	4655	4660	4665	4670	4675	4680	4685	4689
5	4694	4699	4704	4709	4714	4719	4724	4729	4734	4738
6	4743	4748	4753	4758	4763	4768	4773	4778	4783	4787
7	4792	4797	4802	4807	4812	4817	4822	4827	4832	4836
8	4841	4846	4851	4856	4861	4866	4871	4876	4880	4885
9	4890	4895	4900	4905	4910	4915	4919	4924	4929	4934
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983
1	4988	4993	4998	5002	5007	5012	5017	5022	5027	5032
2	5036	5041	5046	5051	5056	5061	5066	5071	5075	5080
3	5085	5090	5095	5100	5105	5109	5114	5119	5124	5129
4	5134	5139	5143	5148	5153	5158	5163	5168	5173	5177
5	5182	5187	5192	5197	5202	5207	5211	5216	5221	5226
6	5231	5236	5240	5245	5250	5255	5260	5265	5270	5274
7	5279	5284	5289	5294	5299	5303	5308	5313	5318	5323
8	5328	5332	5337	5342	5347	5352	5357	5361	5366	5371
9	5376	5381	5386	5390	5395	5400	5405	5410	5415	5419
900	95424	95429	95434	95439	95444	95148	95453	95458	95463	95468

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
1	5472	5477	5482	5487	5492	5497	5501	5506	5511	5516
2	5521	5525	5530	5535	5540	5545	5550	5554	5559	5564
3	5569	5574	5578	5583	5588	5593	5598	5602	5607	5612
4	5617	5622	5626	5631	5636	5641	5646	5650	5655	5660
5	5665	5670	5674	5679	5684	5689	5694	5698	5703	5708
6	5713	5718	5722	5727	5732	5737	5742	5746	5751	5756
7	5761	5766	5770	5775	5780	5785	5789	5794	5799	5804
8	5809	5813	5818	5823	5828	5832	5837	5842	5847	5852
9	5856	5861	5866	5871	5875	5880	5885	5890	5895	5899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
1	5952	5957	5961	5966	5971	5976	5980	5985	5990	5995
2	5999	6004	6009	6014	6019	6023	6028	6033	6038	6042
3	6047	6052	6057	6061	6066	6071	6076	6080	6085	6090
4	6095	6099	6104	6109	6114	6118	6123	6128	6133	6137
5	6142	6147	6152	6156	6161	6166	6171	6175	6180	6185
6	6190	6194	6199	6204	6209	6213	6218	6223	6227	6232
7	6237	6242	6246	6251	6256	6261	6265	6270	6275	6280
8	6284	6289	6294	6298	6303	6308	6313	6317	6322	6327
9	6332	6336	6341	6346	6350	6355	6360	6365	6369	6374
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421
1	6426	6431	6435	6440	6445	6450	6454	6459	6464	6468
2	6473	6478	6483	6487	6492	6497	6501	6506	6511	6515
3	6520	6525	6530	6534	6539	6544	6548	6553	6558	6562
4	6567	6572	6577	6581	6586	6591	6595	6600	6605	6609
5	6614	6619	6624	6628	6633	6638	6642	6647	6652	6656
6	6661	6666	6670	6675	6680	6685	6689	6694	6699	6703
7	6708	6713	6717	6722	6727	6731	6736	6741	6745	6750
8	6755	6759	6764	6769	6774	6778	6783	6788	6792	6797
9	6802	6806	6811	6816	6820	6825	6830	6834	6839	6844
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890
1	6895	6900	6904	6909	6914	6918	6923	6928	6932	6937
2	6942	6946	6951	6956	6960	6965	6970	6974	6979	6984
3	6988	6993	6997	7002	7007	7011	7016	7021	7025	7030
4	7035	7039	7044	7049	7053	7058	7063	7067	7072	7077
5	7081	7086	7090	7095	7100	7104	7109	7114	7118	7123
6	7128	7132	7137	7142	7146	7151	7155	7160	7165	7169
7	7174	7179	7183	7188	7192	7197	7202	7206	7211	7216
8	7220	7225	7230	7234	7239	7243	7248	7253	7257	7262
9	7267	7271	7276	7280	7285	7290	7294	7299	7304	7308
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354
1	7359	7364	7368	7373	7377	7382	7387	7391	7396	7400
2	7405	7410	7414	7419	7424	7428	7433	7437	7442	7447
3	7451	7456	7460	7465	7470	7474	7479	7483	7488	7493
4	7497	7502	7506	7511	7516	7520	7525	7529	7534	7539
5	7543	7548	7552	7557	7562	7566	7571	7575	7580	7585
6	7589	7594	7598	7603	7607	7612	7617	7621	7626	7630
7	7635	7640	7644	7649	7653	7658	7663	7667	7672	7676
8	7681	7685	7690	7695	7699	7704	7708	7713	7717	7722
9	7727	7731	7736	7740	7745	7749	7754	7759	7763	7768
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813

* Electrotypes from Carhart's "Field Book for Civil Engineers."

N	0	1	2	3	4	5	6	7	8	9
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813
1	7818	7823	7827	7832	7836	7841	7845	7850	7855	7859
2	7864	7868	7873	7877	7882	7886	7891	7896	7900	7905
3	7909	7914	7918	7923	7928	7932	7937	7941	7946	7950
4	7955	7959	7964	7968	7973	7978	7982	7987	7991	7996
5	8000	8005	8009	8014	8019	8023	8028	8032	8037	8041
6	8046	8050	8055	8059	8064	8068	8073	8078	8082	8087
7	8091	8096	8100	8105	8109	8114	8118	8123	8127	8132
8	8137	8141	8146	8150	8155	8159	8164	8168	8173	8177
9	8182	8186	8191	8195	8200	8204	8209	8214	8218	8223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
1	8272	8277	8281	8286	8290	8295	8299	8304	8308	8313
2	8318	8322	8327	8331	8336	8340	8345	8349	8354	8358
3	8363	8367	8372	8376	8381	8385	8390	8394	8399	8403
4	8408	8412	8417	8121	8426	8430	8435	8439	8444	8448
5	8453	8457	8462	8466	8471	8475	8480	8484	8489	8493
6	8498	8502	8507	8511	8516	8520	8525	8529	8534	8538
7	8543	8547	8552	8556	8561	8565	8570	8574	8579	8583
8	8588	8592	8597	8601	8605	8610	8614	8619	8623	8628
9	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
1	8722	8726	8731	8735	8740	8744	8749	8753	8758	8762
2	8767	8771	8776	8780	8784	8789	8793	8798	8802	8807
3	8811	8816	8820	8825	8829	8834	8838	8843	8847	8851
4	8856	8860	8865	8869	8874	8878	8883	8887	8892	8896
5	8900	8905	8909	8914	8918	8923	8927	8932	8936	8941
6	8945	8949	8954	8958	8963	8967	8972	8976	8981	8985
7	8989	8994	8998	9003	9007	9012	9016	9021	9025	9029
8	9034	9038	9043	9047	9052	9056	9061	9065	9069	9074
9	9078	9083	9087	9092	9096	9100	9105	9109	9114	9118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
1	9167	9171	9176	9180	9185	9189	9193	9198	9202	9207
2	9211	9216	9220	9224	9229	9233	9238	9242	9247	9251
3	9255	9260	9264	9269	9273	9277	9282	9286	9291	9295
4	9300	9304	9308	9313	9317	9322	9326	9330	9335	9339
5	9344	9348	9352	9357	9361	9366	9370	9374	9379	9383
6	9388	9392	9396	9401	9405	9410	9414	9419	9423	9427
7	9432	9436	9441	9445	9449	9454	9458	9463	9467	9471
8	9476	9480	9484	9489	9493	9498	9502	9506	9511	9515
9	9520	9524	9528	9533	9537	9542	9546	9550	9555	9559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
1	9607	9612	9616	9621	9625	9629	9634	9638	9642	9647
2	9651	9656	9660	9664	9669	9673	9677	9682	9686	9691
3	9695	9699	9704	9708	9712	9717	9721	9726	9730	9734
4	9739	9743	9747	9752	9756	9760	9765	9769	9774	9778
5	9782	9787	9791	9795	9800	9804	9808	9813	9817	9822
6	9826	9830	9835	9839	9843	9848	9852	9856	9861	9865
7	9870	9874	9878	9883	9887	9891	9896	9900	9904	9909
8	9913	9917	9922	9926	9930	9935	9939	9944	9948	9952
9	9957	9961	9965	9970	9974	9978	9983	9987	9991	9996
1000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039

* Electrotypes from Carhart's "Field Book for Civil Engineers."

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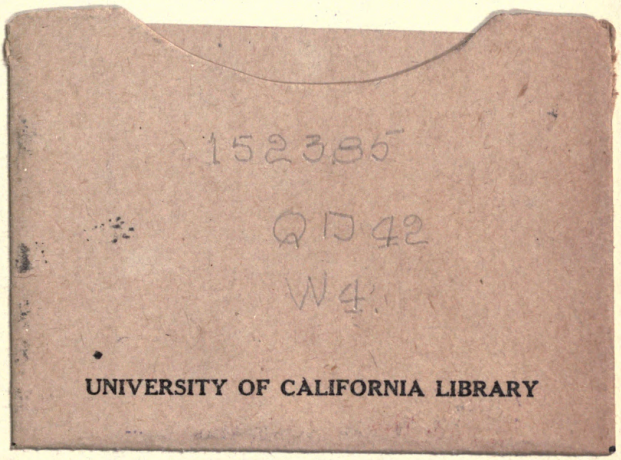
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